BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 1, January 1975

ALGEBRAS OF ANALYTIC FUNCTIONS ON DEGENERATING RIEMANN SURFACES

BY RICHARD ROCHBERG[1]

Communicated by Richard Goldberg, September 30, 1974

I. **Introduction.** By a Riemann surface we mean a finite bordered Riemann surface. For a Riemann surface S denoted by $A(S)$ the supremum normed Banach algebra of functions continuous on S and analytic on the interior of S. For any two Banach spaces A and B define $d(A, B) = \log \inf \{\|T\| \, \|T^{-1}\|;\ T$ a continuous invertible linear map of A onto $B\}$. For S_1 and S_2 homeomorphic Riemann surfaces define $d(S_1, S_2) = d(A(S_1), A(S_2))$. It is known [7] that d defines a metric on $R(S_1)$, the Riemann space of S_1, the space of conformal equivalence classes of Riemann surfaces homeomorphic to S_1, and that the topology induced by this metric is the same as that induced by the Teichmüller metric. The metric space $(R(S_1), d)$ is not complete. In this note we present properties of the ideal elements that are introduced in forming the completion of the metric space $(R(S_1), d)$. Proofs of these and related results will appear in a later publication.

The main result is that the new elements are connected degenerate Riemann surfaces. In fact, the results presented strongly suggest (but do not prove) that the completion of $(R(S_1), d)$ is formed by adjoining to $R(S_1)$ exactly those elements obtained by "pinching to a point" of closed noncontractible curves on surfaces in $R(S_1)$.

On an informal geometric level these results are related to results on degeneration of compact surfaces [4] and results on boundary points of Teichmüller space [1], [2], [5].

II. **An example.** The following example illustrates many of the phenomena described in Theorem 2. For $0 < r < 1$ let $S_r = \{r \leqslant |z| \leqslant 1\}$. Let $A_r = A(S_r)$. Let S_0 be two closed disks with their centers identified, and let A_0 be the algebra of continuous functions on S_0 which are "analytic" on the interior.

AMS (MOS) subject classifications (1970). Primary 30A98, 46J15, 32G15.

[1]This research supported in part by NSF grant GP 34628.