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A COMPARATIVE STUDY OF THE ZEROS OF DIRICHLET L-FUNCTIONS

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We give a comparative study of the zeros of Dirichlet L-functions. Details will appear later.

1. Let χ_1 and χ_2 be distinct primitive characters of the same modulus q, and let $L(s, \chi_i)$, for $i = 1, 2$, be the corresponding Dirichlet L-functions. It is quite natural to guess that $L(s, \chi_1)$ and $L(s, \chi_2)$ have no coincident zero. In other words even a single zero will determine a Dirichlet L-function, or more generally, a "zeta-function". To be more precise, we call ρ a coincident zero of $L(s, \chi_1)$ and $L(s, \chi_2)$ if $L(\rho, \chi_1) = L(\rho, \chi_2) = 0$ with the same multiplicities. And we call ρ a noncoincident zero if it is not coincident. Then we can show

THEOREM 1. *Let χ_1 and χ_2 be distinct primitive characters of the same modulus. Then a positive proportion of the zeros of $L(s, \chi_1)$ and $L(s, \chi_2)$ are noncoincident.*

Next, it is quite natural to guess that the distribution of the zeros of $L(s, \chi_1)$ and $L(s, \chi_2)$ are independent. To state our results, let $\gamma_n(\chi)$ be the ordinate of the nth zero of $L(s, \chi)$ such that $0 \leqslant \gamma_n(\chi) \leqslant \gamma_{n+1}(\chi)$. Further we define $\gamma_n(\chi_1) \leqslant \gamma_m(\chi_2)$ if $\gamma_n(\chi_1) < \gamma_m(\chi_2)$, and $\gamma_n(\chi_1) \leqslant \gamma_m(\chi_2) \leqslant \gamma_{n+1}(\chi_1) \leqslant \gamma_{m+1}(\chi_2) \leqslant \cdots$ if $\gamma_n(\chi_1) = \gamma_{n+1}(\chi_1) = \cdots = \gamma_m(\chi_2) = \gamma_{m+1}(\chi_2) = \cdots$. Then we get

THEOREM 2. *Under the same hypothesis as above, for a positive proportion of $\gamma_n(\chi_1)$'s, there does not exist a $\gamma(\chi_2)$ for which $\gamma_n(\chi_1) \leqslant \gamma(\chi_2) \leqslant \gamma_{n+1}(\chi_1)$.*

Further we define $\Delta_n(\chi_1, \chi_2)$ *to be* $n - m$ *if* $\gamma_m(\chi_1) \leqslant \gamma_n(\chi_2) \leqslant$

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