BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 1, January 1975

CLOSED ALGEBRAS OF SMOOTH FUNCTIONS

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Communicated by I. Kaplansky, May 28, 1974

Introduction. In this note we announce sufficient conditions for an algebra to be a subalgebra of $C^\infty(M)$ for some smooth manifold-without-boundary M. In fact, we are able to determine when M is compact and, more generally, when M carries a Riemannian structure. We maintain the notation and terminology used in [5] and [7]. In addition, $\mathfrak{m}_p$ will denote the unique maximal ideal in the stalk $\mathcal{A}_p$. We assume throughout this note that A is a geometrically homogeneous, harmonic, strongly semisimple, $\mathbf{R}$-algebra with identity. We also assume that A is strongly regular and note that, as a consequence, $\hat{f}$ is a continuous real-valued function on $\Gamma(A)$, for each $f \in A$ [1]. For the sake of brevity, we call an algebra satisfying the above conditions smooth.

Results. If $\mathfrak{m}_p$ is an $\mathcal{A}_p$-module of finite type, then we set $n \cdot \dim_A(M_p)$ equal to the minimal number of generators required for $\mathfrak{m}_p$.

DEFINITION 1. If there exists a positive integer k such that for each $M_p \in \mathfrak{M}(A)$, $n \cdot \dim_A(M_p) = k$, then we say that A has finite naive dimension k, expressed by $n \cdot \dim(A) = k$.

If $\sigma \in H^0(U, \mathcal{A})$, then by $[\sigma](p)$ we mean $[\sigma(p)] \in \mathfrak{m}_p/\mathfrak{m}_p^2$, where $p \in U$.

DEFINITION 2. A is said to be locally framed if for each $M_p \in \mathfrak{M}(A)$ there exists a local unit e_p at M_p, a relatively compact open neighborhood U of p with $p \in \bar{U} \subset u(e_p) \subset \Gamma(A)$, and sections $\sigma_1, \cdots, \sigma_k \in H^0(\Gamma(A), \mathcal{A})$ such that the family

$$([\sigma_1 - \hat{\sigma}_1(q)e_p](q), \cdots, [\sigma_k - \hat{\sigma}_k(q)e_p](q))$$

spans $\mathfrak{m}_q/\mathfrak{m}_q^2$, where $q \in \bar{U}$ and $k = n \cdot \dim(A)$. The sections $\sigma_1|_{\bar{U}}, \cdots,$

AMS (MOS) subject classifications (1970). Primary 26A24, 26A93, 46E25, 50A20, 54H10.

Key words and phrases. Geometrically homogeneous algebra, harmonic algebra, strongly regular algebra, local unit, module of finite type, projective module, derivation, smooth manifold, Riemannian metric.

[1]The author's graduate study is currently supported by an NSF Traineeship.