BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 1, January 1975

SOME SUBALGEBRAS OF $L^\infty(T)$ DETERMINED BY THEIR MAXIMAL IDEAL SPACES

BY T. WEIGHT

Communicated by Paul R. Halmos, June 18, 1974

1. **Introduction.** Sarason [4], [5] has shown, by using the notions of asymptotic multiplicity and vanishing mean oscillation, that $H^\infty(T) + C$ is determined by its maximal ideal space. In this note we announce a generalization of this result to include various superalgebras of $H^\infty(T) + C$. As intermediate steps, we develop localized notions of asymptotic multiplicity and VMO.

2. **Definitions and notation.** (a) Let

$$G_{n,\lambda} = \{z\colon 1 - 1/n < |z| < 1,\ |\arg z - \arg \lambda| < 1/n\}$$

for $\lambda \in T$, $n = 1, 2, \cdots$. For a closed subalgebra A of $L^\infty(T)$ containing $H^\infty(T)$, the Poisson integral is said to be *asymptotically multiplicative* on A at λ if, for $f, g \in A$, $\epsilon > 0$, there exists an N such that $|\hat{f}(z)\hat{g}(z) - \widehat{fg}(z)| < \epsilon$ for $z \in G_{n,\lambda}$ for all $n \geqslant N$.

(b) Now let I be an arc on T; we define θ_I and r_I such that

(i) $e^{i\theta_I}$ is the center of I, and

(ii) $r_I = 1 - \pi m(I)$.

Now we define a collection of arcs $J_{n,\lambda} = \{\text{subarcs } I \text{ of } T\colon r_I e^{i\theta_I} \in G_{n,\lambda}\}$, and for $f \in L^1(T)$ we define

$$M_{n,\lambda}(f) = \operatorname*{Sup}_{I \in J_{n,\lambda}} \frac{1}{m(I)} \int_I |f - f_I|\, dm.$$

We say that $f \in \mathrm{VMO}_\lambda$ if $f \in \mathrm{BMO}$ and $\lim_{n\to\infty} M_{n,\lambda}(f) = 0$. See [4] for a definition and discussion of BMO.

(c) Let $E \subseteq T$; then $L_E^\infty(T)$ will denote the set of functions in $L^\infty(T)$ which can be extended continuously on the set E. When E is a singleton, say $E = \{\lambda\}$, $L_E^\infty(T)$ will be denoted $L_\lambda^\infty(T)$. In case E is σ-compact, it is known [2] that $H^\infty(T) + L_E^\infty(T)$ is a closed algebra.

AMS (MOS) subject classifications (1970). Primary 46J10, 30A76, 30A78.