

AN ALGORITHM FOR THE TOPOLOGICAL DEGREE OF A MAPPING IN n -SPACE¹

BY FRANK STENGER

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1. Introduction. In this paper we announce a new formula for computing the topological degree $d(F, P, \theta)$, where $F = (f^1, \dots, f^n)$ is a vector of real continuous functions mapping a polyhedron P in R^n into R^n , and θ is the zero vector in R^n .

Let $A = [a_{ij}]$ be an $n \times n$ real matrix, and let A_i denote the i th row of A . We use the convenient notation $\Delta_n(A_1, \dots, A_n)$ for the determinant of A , and $|A_i| \equiv (a_{i1}^2 + \dots + a_{in}^2)^{1/2}$ for the Euclidean norm of A_i .

Let X_0, X_1, \dots, X_q denote $q + 1$ points in R^n , where $q \leq n$, such that the vectors $X_i - X_0, i = 1, 2, \dots, q$, are linearly independent. A q -simplex with vertices at X_0, \dots, X_q is defined by

$$S_q(X_0, \dots, X_q) \equiv \left\{ X \in R^n : X = \sum_{i=0}^q \lambda_i X_i, \lambda_i \geq 0, \sum_{i=0}^q \lambda_i = 1 \right\}.$$

We denote by $[X_0 \dots X_q]$ the oriented q -simplex, defined as in [2]. For example, if $q = n$, then $[X_0 \dots X_q] = [X_0 \dots X_n]$ is said to be positively (negatively) oriented in R^n if $\Delta_{n+1}(Z_0, \dots, Z_n) > 0$ (< 0), where $Z_i = (1, X_i)$.

Let P be a connected, n -dimensional closed polyhedron represented as a "sum" of m' positively oriented n -simplexes in the form

$$(1.1) \quad P = \sum_{j=1}^{m'} [X_0^{(j)} \dots X_n^{(j)}]$$

such that the intersection of any two of the simplexes has zero n -dimensional volume.

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