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ANALYSIS AND SYNTHESIS FOR THE POINT AND UNITARY SPECTRA. II

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This is a direct sequel to Part I [1], continuing presentation of results from a portion of [2]. The terminology and notation of [1] are retained. Further, we will denote by E_λ the subspace $\mathrm{Ker}(T - \lambda I)$ of eigenvectors of the operator T belonging to λ; by $R(\lambda, T)$, the resolvent $(\lambda I - T)^{-1}$ of T. The notation for "closed linear span of $\cdots$" will be "$L(\cdots)$".

This section gives solutions of the problem of analysis-synthesis for complete operators whose point spectrum $\sigma_p(T)$ is all or mostly in the circle Γ and which are close to being isometric. We will call an operator 'Γ-complete' in case $L(E_\lambda\colon \lambda \in \Gamma) = X$. Various conditions of closeness to isometricity are considered: 'strong ergodicity' ($K(T) = \sup_{n\geq 0} \|T^n\|_X < +\infty$); '$C$-ergodicity'

$$\left(\sup_{\lambda\in\Gamma, n\geq 0} \left\| \frac{1}{n+1} \sum_{k=0}^{n} T^k\lambda^k \right\|_X < +\infty\right);$$

'A-ergodicity' ($\sup_{|\lambda|>1} \|R(\lambda, T\|_X(|\lambda| - 1) < +\infty$); and others. Any of these conditions permits application of an ergodic theorem to ensure existence of a "spectral projector" $P_\lambda = \lim_{r\to 1+0}(r-1)R(r\lambda, T)$ onto the eigenspace E_λ along all the other such subspaces: $P_\lambda P_\mu = 0$ if $\lambda \neq \mu$. This makes it possible to associate to every vector $x \in X$ its formal Fourier series

$$(*) \qquad x \sim \sum_{\lambda\in\sigma_p(T)} P_\lambda x.$$

The study of Γ-complete operators leads to a far-reaching analogy with the study of the group of rotations of Γ acting in the spaces of classical analysis (L^p, l^p, Orlicz spaces, etc.). It lies near at hand to supplement the definitions given in [1]: setting $E(x) = L(T^n x\colon n \geq 0)$, we will say that the

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