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ON THE DISTRIBUTION OF THE ZEROS OF THE RIEMANN ZETA FUNCTION IN SHORT INTERVALS

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Here we are concerned with the distribution of the zeros of the Riemann zeta function $\zeta(s)$ in short intervals in the vertical direction. We establish a mean value theorem for the number of the zeros in short intervals and derive several consequences from it, in particular, about the difference between the ordinates of the zeros and properties of uniform distribution of the zeros. Details will appear later.

We start from the Riemann-von Mangoldt formula for the number $N(t)$ of the zeros of $\zeta(s)$ in $0 < \operatorname{Im} s \leqslant t, 0 < \operatorname{Re} s < 1$, where the possible zeros on $\operatorname{Im} s = t$ are counted with weight one half:

$$N(t) = L(t) + S(t) \quad \text{for} \quad t > t_0,$$

where

$$L(t) = \frac{t}{2\pi} \log t - \frac{1 + \log 2\pi}{2\pi} t + \frac{7}{8} + O\left(\frac{1}{t}\right).$$

Here

$$S(t) = (1/\pi) \arg \zeta(\tfrac{1}{2} + it),$$

where $\arg \zeta(\frac{1}{2} + it)$ is defined by continuous variation on the half line $\sigma + it$, $\sigma \geqslant \frac{1}{2}$ starting with the value zero if t is not the ordinate of a zero of $\zeta(s)$. If the path crosses a zero, we put

$$\arg \zeta(\tfrac{1}{2} + it) = \tfrac{1}{2}\{\arg \zeta(\tfrac{1}{2} + i(t + 0)) + \arg \zeta(\tfrac{1}{2} + i(t - 0))\}$$

(cf. [5, 9.2.]). Then the number

$$\Delta_h N(t) \equiv N(t + h) - N(t) = L(t + h) - L(t) + S(t + h) - S(t)$$

of the zeros of $\zeta(s)$ in a short interval $t < \operatorname{Im} s \leqslant t + h$ essentially depends

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