BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 1, January 1975

HARMONIC FORMS AND RIESZ TRANSFORMS FOR RANK ONE SYMMETRIC SPACES

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Communicated by Hans Weinberger, June 11, 1974

We study harmonic forms on a noncompact rank one symmetric space M; that is, differential forms satisfying the equations $d\omega = 0$, $\delta\omega = 0$. We define "Hardy spaces" $\mathbf{H}^p$ of harmonic forms on M and study their boundary behavior. Fractional and singular integral operators are introduced on an Iwasawa group $\overline{N}$ of M, and used to characterize the boundary values of forms in $\mathbf{H}^p$, setting up an explicit isomorphism between these spaces and the ordinary L^p spaces on $\overline{N}$. In this sense, these operators play a similar role to that of the Riesz transforms on Euclidean spaces and compact Lie groups associated to the "conjugate systems" of harmonic functions studied by Coifman, Stein, and Weiss [1], [7].

1. **Some vector fields on** M. Let G be the identity connected component of the group of isometries of M; fix an Iwasawa decomposition $G = KAN$ of G, and let $\overline{N} = \theta N$, where θ is the Cartan involution of G associated to K. Let $\mathfrak{g}$, $\mathfrak{k}$, $\mathfrak{n}$, $\overline{\mathfrak{n}}$, $\mathfrak{a}$ be the Lie algebras of the groups G, K, N, $\overline{N}$, A. Now define a right-action τ of the solvable group $\overline{S} = \overline{N}A$ on $M = G/K$ as follows: since $G = \overline{S}K$, each $x \in M$ can be written uniquely as $x = s \cdot o$, where $o = \{K\}$, $s \in \overline{S}$; then for $s' \in \overline{S}$ let $\tau(s')(s \cdot o) = ss' \cdot o$. For $X \in \overline{\mathfrak{s}}$, considered as a left invariant vector field on $\overline{S}$, define a vector field $\widetilde{X}$ on M by $\widetilde{X}_{\overline{n}a\circ o} = \tau_*(\mathrm{Ad}\,(a^{-1})X)$, $\overline{n} \in \overline{N}$, $a \in A$, where τ_* denotes the infinitesimal action of $\overline{\mathfrak{s}}$ on M induced by τ. Since the action τ is free, $X \longrightarrow \widetilde{X}$ maps a basis of $\overline{\mathfrak{s}}$ onto an everywhere defined frame of vector fields on M. Moreover, $[X, Y]^\sim = [\widetilde{X}, \widetilde{Y}]$ whenever X and Y are both in $\overline{\mathfrak{n}}$ and $[\widetilde{X}, \widetilde{Y}] = 0$ if $X \in \mathfrak{a}$. Note that the integral curves of $\widetilde{X}$, $X \in \mathfrak{a}$ are geodesics in M which are orthogonal to the family of submanifolds $\overline{N}a \cdot o$, $a \in A$.

AMS (MOS) subject classifications (1970). Primary 43A85, 53C35; Secondary 35H05.

Key words and phrases. Harmonic forms, Lie groups, fundamental solution, singular and fractional integrals.

[1]Partially supported by NSF grant GP 29704 A3.

[2]Partially supported by NSF grant GP 28448.