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COMMUTATIVE SUBALGEBRA OF $L^1(G)$ ASSOCIATED WITH A SUBELLIPTIC OPERATOR ON A LIE GROUP G

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1. **Introduction.** Let G be a Lie group and LG its Lie algebra regarded as the space of differential operators of the first order which commute with the right translations. If $X_1, \cdots, X_n$ is a basis of LG, then the operator $L = X_1^2 + \cdots + X_n^2$ is called a laplacian on G. In [4] the commutative Banach $*$-subalgebra of $L^1(G)$ generated by the fundamental solution of the heat equation $(\partial/\partial t - L)u = 0$ was studied, and in case of compact extensions of nilpotent groups it proved to be useful in studying spectral properties of L on various $L^p(G)$ spaces, as well as in proving tauberian Wiener theorems concerning Gauss and Poisson integrals. In [6] and [9] a powerful method of singular integrals on the class of nilpotent Lie groups admitting one-parameter groups of dialations was developed. In [1] and [2] Folland and Stein studied the relation of these to certain subelliptic operators on the Heisenberg group. The idea is that in various important cases, although for a given one-parameter group of dialations $\{\delta_s\}$, $s > 0$, of G there is no basis in LG such that $\delta_{s*}L = s^\lambda L$ where λ is a scalar, there exists a set of generators $X_1, \cdots, X_k$ of the Lie algebra LG such that $\delta_{s*}X_j = sX_j$, $j = 1, \cdots, k$. Let

$$L = X_1^2 + \cdots + X_k^2. \tag{1}$$

Then, of course,

$$\delta_{s*} L = s^2 L. \tag{2}$$

The fact that $X_1, \cdots, X_k$ generate LG as a Lie algebra implies that L is a subelliptic operator. Using this fact we shall construct the Gauss and Poisson kernels for the operator L, and via a study of the subalgebra of $L^1(G)$ generated by these, we obtain the equality of the spectra of L on various $L^p(G)$ spaces as well as the corresponding tauberian Wiener theorems. More-

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