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L_2-REPRESENTATIONS AND A PLANCHEREL-TYPE THEOREM FOR PARABOLIC SUBGROUPS

BY FREDERICK W. KEENE

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Let G be a semisimple Lie group with Iwasawa decomposition $G = KAN$. In this note we give a precise condition for the existence of square-integrable representations of the nilpotent subgroup N. In that case we write down a Plancherel formula for the solvable subgroup AN. Full details and complete proofs will appear in a later paper.

These results are in essence part of the author's doctoral dissertation [1]. He would like to thank Professor Joseph A. Wolf for his patient advice and encouragement.

I. **L_2-representations of the nilpotent subgroup** N. Let N be a unimodular locally compact group. Let π be an irreducible unitary representation of N on a Hilbert space $H(\pi)$. Then π is square-integrable (or L_2) if there are nonzero vectors x_1 and x_2 in $H(\pi)$ such that

$$\int_{N/Z} |(\pi(s)x_1, x_2)|^2 \, d\mu(s) < \infty$$

where Z is the center of N and $d\mu(s)$ denotes integration over N/Z with respect to a Haar measure μ on N/Z.

If N is a connected simply connected nilpotent Lie group with Lie algebra $\mathfrak{n}$, let Z and $\mathfrak{z}$ be the respective centers on N and $\mathfrak{n}$. Let $\mathfrak{n}^*$, $\mathfrak{z}^*$ be the respective linear duals of $\mathfrak{n}$, $\mathfrak{z}$. Define an alternating bilinear form b_f on $\mathfrak{n}$ by $b_f(x, y) = f([x, y])$ for $f \in \mathfrak{n}^*$ and $[\ ,\ ]$ the multiplication for $\mathfrak{n}$. If $f \in \mathfrak{z}$, we can extend f trivially to $\mathfrak{n}$ and define b_f on $\mathfrak{n}/\mathfrak{z}$. Moore and Wolf [3] have shown the following:

PROPOSITION 1. *N has L_2-representations if and only if there exists an $f \in \mathfrak{z}^*$ such that b_f is nondegenerate on $\mathfrak{n}/\mathfrak{z}$.*

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