BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 1, January 1975

APPROXIMATION THEOREMS, C^∞ CONVEX EXHAUSTIONS AND MANIFOLDS OF POSITIVE CURVATURE[1]

BY R. E. GREENE AND H. WU

Communicated by S. S. Chern, June 25, 1974

In this note, we announce several approximation theorems on Riemannian manifolds as well as some of their consequences. First recall the relevant definitions. Let M be a Riemannian manifold. A function f: $M \longrightarrow \mathbf{R}$ is called *convex* iff its restriction to each geodesic is a convex function of one variable. A function f on M is called *strictly convex* iff given any compact $K \subset M$, there exists an $\epsilon > 0$ such that for every geodesic $\tau(t)$ parameterized by arc-length and defined on $(-s,s)$ with $\tau(0) \in K$, $f(\tau(s)) + f(\tau(-s)) - 2f(\tau(0)) > \epsilon s^2$ for all $s \in (0, \epsilon)$. A function f is *subharmonic* iff it is everywhere a subsolution of the Dirichlet problem, i.e. if B is a sufficiently small geodesic ball and u is a harmonic function such that $u = f$ on the boundary of B, then $u \geqslant f$ everywhere in B. If f is C^2, f is subharmonic iff $\Delta f \geqslant 0$, where $\Delta =$ the Riemannian metric Laplacian. For a $C^2 f$, we define f to be *strictly subharmonic* iff $\Delta f > 0$. If f is merely continuous, we say f is strictly subharmonic iff at each $x \in M$, there exists a C^2 strictly subharmonic f_0 near x such that $f - f_0$ is subharmonic near x.

Suppose M is a complex manifold, not necessarily equipped with a Riemannian metric. A real-valued C^2 function f on M is called *strictly plurisubharmonic* iff $d'd''f$ is a positive definite Hermitian form at each point. For a continuous function f, we say f is *strictly plurisubharmonic* iff at each point $x \in M$, one can find a C^2 strictly plurisubharmonic function f_0 near x such that $f - f_0$ is plurisubharmonic near x.

In the following, S will denote any one of the following subsets of the ring of real-valued functions on M: (A) convex functions, (B) continuous subharmonic functions, (C) continuous plurisubharmonic functions on a complex manifold, (D) continuous plurisubharmonic functions on a complex

---

AMS (MOS) subject classifications (1970). Primary 53C20, 57D12; Secondary 53C55, 32F05, 31C05.

[1]Research partially supported by the National Science Foundation.