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CONICAL DISTRIBUTIONS FOR RANK ONE SYMMETRIC SPACES

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Let $X = G/K$ be a symmetric space of noncompact type, where G is a connected semisimple Lie group with finite center, and K is the compact part of an Iwasawa decomposition $G = KAN$ of G. Let M (M') be the centralizer (normalizer) of A in K. Then the space Ξ of all horocycles of X can be identified with G/MN or $(K/M) \times A$ [1, p. 8]. The set of all smooth functions with compact supports on Ξ endowed with the customary topology is denoted by $\mathcal{D}(\Xi)$. Its dual $\mathcal{D}'(\Xi)$ consists of all distributions on Ξ. Let W be the Weyl group M'/M and $\mathfrak{A}_C^*$ be the complex dual of $\mathfrak{A}$, the Lie algebra of A.

DEFINITION [1, p. 65]. A distribution $\Psi \in \mathcal{D}'(\Xi)$ is said to be *conical* if (i) Ψ is MN-invariant, (ii) Ψ is an eigendistribution of every G-invariant differential operator on Ξ.

As is readily seen, this definition is parallel to that of spherical functions on X. On this basis S. Helgason made the conjecture that the set of all conical distributions can be parametrized by $W \times \mathfrak{A}_C^*$, and he also established it in various cases [1, Chapter III, §4]. Our purpose here is to complete its verification in case X has rank one.

Now for each $a \in A$, there is a map $\sigma(a)$ of Ξ defined by $\sigma(a)(gMN) = gaMN$. This gives rise to a corresponding action $\Psi \longmapsto \Psi^{\sigma(a)}$ on the space $\mathcal{D}'(\Xi)$. If $\lambda \in \mathfrak{A}_C^*$, let $\mathcal{D}'_\lambda = \{\Psi \in \mathcal{D}'(\Xi) \mid \Psi^{\sigma(a)} = e^{-(i\lambda+\rho)\log a}\Psi, \forall a \in A\}$, where ρ is half the sum of all positive restricted roots, counting multiplicity, and $\log : A \longrightarrow \mathfrak{A}$ is the inverse of the exponential map. The space $\mathcal{D}'_\lambda$ consists of the joint eigenspaces of the G-invariant differential operators on Ξ [1, p. 69]. So an element $\Psi \in \mathcal{D}'(\Xi)$ is conical iff it is (i) MN-invariant, and (ii) belongs to some $\mathcal{D}'_\lambda$. Next we recall some constructions from [1, Chapter III, §4]. For each $s \in M'/M$, choose an $m_s \in M'$ in the

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