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GROUP ACTIONS AND CURVATURE

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1. **Introduction.** The purpose of this note is to outline a proof of the following result: Any isometric action of a compact Lie group G on a 1-connected, compact Riemannian manifold M whose curvature tensor R is sufficiently close to the curvature tensor R_0 of the standard sphere S^n of the same dimension is equivalent to an isometric action of G on S^n.

We measure the proximity of R and R_0 in terms of the eigenvalues of the curvature transformation $R: V\wedge V\to V\wedge V$, where $V=T(M)$. A Riemannian manifold M is called *strongly δ-pinched* if the eigenvalues λ of the curvature transformation at all points of M satisfy the condition $\delta<\lambda\leqq 1$.

2. **Statement of results.** The main result is as follows:

THEOREM. *There exists a $\delta_0<1$, such that for any 1-connected, compact, strongly δ-pinched n-dimensional Riemannian manifold M, and any compact Lie group G the following holds:*

If $\delta>\delta_0$ and $\mu: G\times M\to M$ is an isometric action of G on M, then

(1) *there exists a diffeomorphism $F: M\to S^n$;*

(2) *there exists a homomorphism $\omega: G\to O(n+1)$ such that*

(3) *$\omega(g)=F\circ\mu(g,\cdot)\circ F^{-1}$ for all $g\in G$.*

The following two corollaries are immediate consequences.

COROLLARY 1. *Any compact, strongly δ-pinched Riemannian manifold M with $\delta>\delta_0$ is diffeomorphic to a space of constant curvature* 1.

Together with Wolf's [4] classification of manifolds with constant curvature 1, this corollary gives a classification up to diffeomorphism of compact, strongly δ-pinched Riemannian manifolds with $\delta>\delta_0$. In addition, the isometry group of such a manifold is isomorphic to a subgroup of the isometry group of the corresponding manifold with constant curvature.

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