

## BOOK REVIEW

*Topological Riesz spaces and measure theory*, by D. H. Fremlin, Cambridge University Press, New York, 1974, xiv+266 pp., \$17.50

A *Riesz space*, or *vector lattice*, is a partially ordered real vector space which, as a partially ordered set, is a lattice. Many of the most important spaces of functions occurring in analysis, and in particular in measure and integration theory, are Riesz spaces, and it is therefore not surprising that the abstract theory of Riesz spaces should play a significant role in the study of such function spaces. In fact, it is equally true that the Riesz space structure of function spaces has had a profound influence on the development of the abstract theory. The purpose of D. H. Fremlin's book is to identify those concepts in the abstract theory of Riesz spaces which have particular relevance to the older discipline of measure theory, and to show how the corresponding aspects of measure theory may be illuminated using the techniques of functional analysis on Riesz spaces.

The book is primarily addressed to functional analysts, and a basic knowledge of functional analysis, including the rudiments of the theory of topological vector spaces, is assumed. A knowledge of the basics of abstract measure theory will also be of considerable help to the reader, although strictly speaking this is formally unnecessary. Otherwise, no specialized knowledge is required and the book will consequently be accessible to a reasonably well-educated graduate student.

The contents fall loosely into four sections. Chapters 1–3 give an account of the basic theory, both algebraic and topological, of Riesz spaces. This is followed by three chapters in which the theory is put to work to give a development of abstract measure theory, after which comes a chapter on the representation by integrals of linear functionals on function spaces. The book concludes with a chapter on weakly compact sets in Riesz spaces.

The basic algebraic theory of Riesz spaces is given in Chapter 1. Firstly, the fundamental results connecting the algebraic and lattice structures are obtained, and further important concepts (e.g. Dedekind completeness, quotient spaces, Archimedean spaces) are introduced. The discussion then turns to the various classes of linear mappings between Riesz spaces, and it is here that the real force of the axioms of a Riesz space become apparent. The most important spaces of mappings associated with a Riesz space  $E$  are the two (order) dual spaces  $E^\sim$  and  $E^\times$ . The space  $E^\sim$  consists of those linear functionals on  $E$  which are expressible as the