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A SPECTRAL DECOMPOSITION THEOREM FOR CERTAIN HARMONIC ALGEBRAS

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Introduction. Let K be a simple ring with identity and let A be a harmonic K-algebra with identity, where neither K nor A is assumed to be commutative. If one denotes the set of maximal ideals in A by $\mathrm{Max}(A)$, then A is strongly semisimple iff $S(A) = \bigcap_{M \in \mathrm{Max}(A)} M = (0)$. We assume that A is strongly semisimple and note that this implies that A is Jacobson semisimple. One may equip $\mathrm{Max}(A)$ with the hull-kernel topology, and we denote this space by $\max(A) = (\mathrm{Max}(A), \tau)$. We index $\mathrm{Max}(A)$ by the points of $\max(A)$; viz., if $p \in \max(A)$ then $M_p \in \mathrm{Max}(A)$ is the ideal corresponding to p. Since A is harmonic, the space $\max(A)$ is a locally compact Hausdorff space [8] and, as usual, since A has an identity, $\max(A)$ is compact. Teleman [8] has also shown that there exists a plastic, semisimple sheaf of local algebras such that $A \cong H^0(\max(A), \mathcal{A})$, where $H^0(\max(A), \mathcal{A})$ is the K-algebra of global sections of $\mathcal{A}$. More generally, if B is a strongly semisimple harmonic ring, then B is isomorphic to a subring of $H^0(\max(B), \mathcal{B})$. However, this representation is too general for the applications we have in mind, as the elements of A may take values in different simple rings. In addition, one of the characteristic features of the representation of rings by sections is that the topology of the base space can be extremely unmanageable [4]. In this note, we state sufficient conditions under which we are able to extract from Teleman's representation a second representation of A as a ring of K-valued functions on a homogeneous, locally compact Hausdorff base space. It is obvious that, under mild restrictions, these conditions are also necessary.

RESULTS. We denote the group of K-algebra automorphisms of A by $\mathrm{Aut}_K(A)$.

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