## SOLVABILITY ON MANIFOLDS BY QUADRATURES PERMITTING ONLY INTEGRALS

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Let M be a connected  $C^{\infty}$  manifold, and let  $\rho: \widetilde{M} \to M$  be the universal covering map. Choose a base point  $\widetilde{x}_0 \in \widetilde{M}$ , and write  $x_0 = \rho \widetilde{x}_0$ . The fundamental group  $\pi_1(M)$  is assumed to be finitely generated.

Let A be a subcomplex of the de Rham complex  $\Lambda(M)$  satisfying the conditions:

(a) The subcomplex A is closed under the exterior product.

(b) The inclusion  $A \subset \Lambda(M)$  induces an isomorphism  $H(A) \approx H(\Lambda(M))$ . Write  $F_0 = \rho^* A^0$  and  $\Omega = \rho^* A^1$ . If  $w \in A^1$  is a closed 1-form on M, then the integral  $\int_{\widetilde{X}_0} \rho^* w$  is a function on  $\widetilde{M}$  and can be regarded as a multivalued function on M. All such integrals together with 1 span a vector space  $F_1$  of functions on  $\widetilde{M}$  such that  $F_0 \subset F_1$ . For  $r \ge 1$ , define  $F_{r+1}$  to be the vector space of functions spanned by  $F_r$  and all  $\int_{\widetilde{X}_0} w$ , w being closed 1-forms belonging to the subspace  $F_r\Omega$  of  $\Lambda^1(\widetilde{M})$ . It turns out that  $\widetilde{F} = \bigcup_{r\ge 0} F_r$  is an algebra of functions on  $\widetilde{M}$ .

Recall that the lower central series of a group G consists of commutator subgroups  $G_r$ ,  $r \ge 1$ , defined by  $G_1 = G$  and  $G_{r+1} = [G_r, G]$ ,  $r \ge 1$ . The lower central series is said to stabilize modulo torsion if  $G_r/G_{r+1}$ is finite for r sufficiently large. A group G is said to be torsion free residually nilpotent if each quotient  $G_r/G_{r+1}$  is torsion free and if  $\bigcap G_r = \{e\}$ .

The purpose of this note is to announce the next results, which will be proved in detail elsewhere.

THEOREM 1. The algebra  $\widetilde{F}$  is finitely generated over F if and only

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