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AN APPROXIMATION PROPERTY CHARACTERIZES ORDERED VECTOR SPACES WITH LATTICE-ORDERED DUALS

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This note announces that a simple approximation property, known to hold in large classes of partially ordered locally convex spaces whose duals are lattice ordered (e.g., [Banach or] Fréchet spaces with closed, normal, generating positive cones possessing the Riesz decomposition property), actually characterizes spaces with lattice-ordered duals. Some very mild and natural assumptions about the relation of the topology and the order have to be made, but these are automatically satisfied in Fréchet spaces with closed normal generating cones. This characterization has important consequences in the duality theory of certain classes of linear operators and in questions regarding approximation of harmonic functions; those consequences will be taken up elsewhere.

Let $E[\mathfrak{T}]$ be a locally convex space, let E' be its dual, and let K be a closed wedge in E, i.e., a closed convex set carried into itself under multiplication by nonnegative scalars; let $K' = -K^0$ denote the wedge dual to K. (In the interesting cases K will be a cone.) K determines a wedge $L(E)^+$ in the algebra $L(E)$ of continuous linear transformations of E into itself, $L(E)^+$ consisting of those transformations that carry K into itself. We shall indicate that an element of one of these spaces belongs to the corresponding wedge by saying that the element is "nonnegative" or "$\geqslant 0$".

The tensor product space $E' \otimes E$ is isomorphic to the ideal in $L(E)$ consisting of linear transformations of finite rank; if $\mathfrak{t} = \Sigma x'_i \otimes x_i \in E' \otimes E$, the corresponding linear transformation is $T_{\mathfrak{t}} = (x \longrightarrow \Sigma\langle x, x'_i\rangle x_i)$(sums indicated in this way are understood to be finite). Abusing notation slightly, let $K' \otimes K = \{\Sigma x'_i \otimes x_i \colon x_i \in K, x'_i \in K'\}$; it is clear that $K' \otimes K$ is a wedge. Let $P = \{T_{\mathfrak{p}} : \mathfrak{p} \in K' \otimes K\}$.

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