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ASYMPTOTIC NONUNIQUENESS OF THE NAVIER-STOKES EQUATIONS IN KINETIC THEORY[1]

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We consider the linearized Boltzmann equation

$$\partial p/\partial t + \xi \cdot \operatorname{grad} p = Qp/\epsilon, \tag{1}$$

whose solution $p = p_\epsilon(t, x, \xi)$, $t > 0, x \in R^3, \xi \in R^3, \epsilon > 0$. Q is the linearized collision operator corresponding to a spherically symmetric hard potential, and ϵ is a parameter which represents the mean free path.

In a series of basic papers, Grad [6], [7], [8] studied the existence and asymptotic behavior of the solution of the initial value problem for (1), where the initial data $p_\epsilon(0^+, x, \xi) = f(x, \xi)$ satisfies mild growth and smoothness conditions. Grad's method begins with the decomposition

$$Q = -\nu + K, \tag{2}$$

where ν is the operator of multiplication by the collision frequency $\nu(\xi)$, a strictly positive function of $|\xi|$, and K is a compact operator on the Hilbert space H_0 of functions $f(\xi)$ which satisfy

$$\langle f, f\rangle \equiv \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int |f(\xi)|^2 \exp(-|\xi|^2/2)\, d\xi < \infty.$$

Using (2), Grad wrote (1) as an integral equation and then derived *a priori* estimates for the solution in the Hilbert space

$$H \equiv L^2(R^6, (1/\sqrt{2\pi})^3 \exp(-|\xi|^2/2)\, dx\, d\xi).$$

Grad also related the asymptotic behavior of p_ϵ to the solutions of the linear Euler and Navier-Stokes equations. Given $f \in H$, define

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