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SOME PROBLEMS OF MATHEMATICS AND SCIENCE[1]

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Introduction. The development of mathematics has often been aided by the use of models from science and technology. There are three main reasons why models help: (i) attention is focused on significant problems; (ii) the intuition is aided in perceiving complex relations; (iii) new concepts are suggested. This paper describes problems arising from models which have interested me. The models come from physics, chemistry, engineering, and economics.

1. **The Dirichlet problem for the wave equation.** We are given two photographs of a vibrating string, one at time $t=0$ and another at a later time $t=\alpha$, as illustrated in Figure 1. Is it possible to determine the state

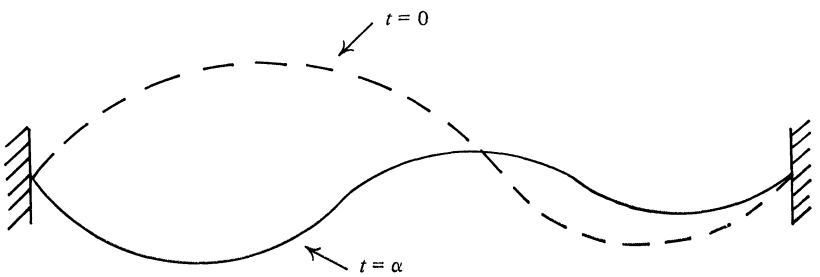

FIGURE 1. The vibrating string

at intermediate times? This puzzle led David Bourgin and me to study the Dirichlet problem for the wave equation. The wave equation is

$$\partial^2 y/\partial x^2 = \partial^2 y/\partial t^2$$

and the region of concern is the rectangle R, $0 \leqq x \leqq 1$, $0 \leqq t \leqq \alpha$. There are solutions of the form $y=\sin(\pi n x)\sin(\pi n t)$ for any integer n. Then if α is a rational number, say $\alpha=m/n$, it is seen that given y at $t=0$ and $t=\alpha$ does not determine y at intermediate times. On the other hand, we found that if α was irrational and y was of class C^2 in the rectangle R, then y was uniquely determined.

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