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INJECTIVE STABILITY FOR K_2 OF LOCAL RINGS

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It has been conjectured for some time **[St2]**, **[D]**, **[S-D]**, **[D-S2]** that for a semilocal ring R, the homomorphisms $\theta_n: K_2(n, R) \to K_2(n+1, R)$, known to be surjective for all $n \geqq 2$ **[St2]**, **[S-D]**, are in fact isomorphisms for $n \geqq 3$. Various special cases have been proved, most notably the difficult theorem of Matsumoto for fields [**Ma**, Corollaire 5.11] and the case of discrete valuation rings **[D-S1]**. Matsumoto also shows that θ_2 is not an isomorphism in general.

In this note we announce the proof of the following theorem, details of which will be published elsewhere. Unexplained notation and terminology is that of **[Mi]**.

THEOREM A. *Let R be a commutative local ring. The homomorphisms* θ_n *are isomorphisms, and consequently,* $K_2(n, R) \approx K_2(R)$ *for any* $n \geqq 3$.

In broad outline, the proof of Theorem A is similar to that of Matsumoto for fields [**Mi**, §12]. The maps θ_n are surjective as $K_2(n, R)$ is generated by the Steinberg symbols $\{u, v\}$, $u, v \in R^*$ [**St2**, Theorem 2.13]. To show that for $n \geqq 3$ the θ_n are injective, the symbol $\{\ ,\ \}$ with values in the group $A = K_2(3, R)$ is used to construct a central extension

$$(*) \qquad 1 \to A \to G_n \to E(n, R) \to 1.$$

Since $\mathrm{St}(n, R)$ is the universal central extension of $E(n, R)$ for $n \geqq 5$, $(*)$ implies the existence of a surjective homomorphism $\mathrm{St}(n, R) \to G_n$ which induces a surjection $K_2(n, R) \to A$ inverse to the surjection

$$K_2(3, R) \to K_2(n, R).$$

Thus Theorem A follows immediately from the construction of the central extensions $(*)$.

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