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GLOBAL BIFURCATION THEOREMS FOR NONCOMPACT OPERATORS

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1. **Introduction.** The first general existence theorem for bifurcation points was obtained by Krasnoselski [**1**]. He considered the equation $u=\lambda Lu+H(\lambda, u)$ in a real Banach space $\mathscr{B}$ where L and H are compact, and H is $o(\|u\|)$ uniformly on each bounded λ interval for small u. In this situation he proved that if λ is a characteristic value of L having odd multiplicity, then $(\lambda, 0)$ is a bifurcation point in $R\times\mathscr{B}$. Much more recently, Rabinowitz [**2**] considered the same problem and, using a Leray-Schauder degree argument, obtained a two-fold alternative for the global behavior of these bifurcation branches.

This paper extends the results of Krasnoselski and Rabinowitz to a much larger class of operator equations. First to be considered is the equation

$$Lu = \lambda u + H(\lambda, u) \tag{1}$$

in a real Hilbert space $\mathscr{H}$, where H is as above and L is selfadjoint (bounded or unbounded). In this case, each isolated eigenvalue of L having odd multiplicity is a bifurcation point possessing a continuous branch. Moreover, an alternative theorem on the global behavior of these branches is obtained.

By use of similar arguments these results for selfadjoint operators are extended to a general class of linear operators in a real Banach space $\mathscr{B}$.

2. **The selfadjoint operators.** In this section all work is in a real Hilbert space $\mathscr{H}$, L is a selfadjoint operator taking $\mathscr{H}$ into $\mathscr{H}$, and $H(\lambda, u)$ is a compact operator taking $R\times\mathscr{H}$ into $\mathscr{H}$ that is $o(\|u\|)$ uniformly on each bounded λ interval for small u.

Let $\mathscr{E}$ denote $R\times\mathscr{H}$ with the product topology. For $\mathscr{V}\subset\mathscr{E}$, a subcontinuum of $\mathscr{V}$ is a subset of $\mathscr{V}$ which is closed and connected in $\mathscr{E}$. The trivial solutions of (1) are the points $(\lambda, 0)$, and all other solutions are called nontrivial. Let $\mathscr{S}$ denote all nontrivial solutions of (1), and let $\mathscr{C}_{\lambda_0}$ denote the maximal subcontinuum of $\mathscr{S}\cup(\lambda_0, 0)$ containing $(\lambda_0, 0)$.

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