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ON A p-ADIC VANISHING THEOREM OF GARLAND

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Let k be any nonarchimedean locally compact field, with (say) q elements in its residue field. Let $\boldsymbol{G}$ be a simple, semisimple algebraic group defined over k, G the group of its k-rational points on $\boldsymbol{G}$. Let l be the k-rank of $\boldsymbol{G}$. Suppose Γ to be a discrete subgroup of G such that $\Gamma\backslash G$ is compact, and V a finite-dimensional vector space over $\boldsymbol{C}$ on which Γ has a unitary representation. In [**4**], Garland has proven that there exists an integer $q(l)$ (depending only on l) such that if $q>q(l)$ then $H^m(\Gamma, V)=0$ for $m\neq 0, l$. Garland's proof is an analogue of the proofs of vanishing theorems for discrete subgroups of real groups, applying a sort of curvature on the Bruhat-Tits complex of G. By an apparently entirely different method, I have been able to remove the residue field restriction and thus to prove

THEOREM 1. *One has $H^m(\Gamma, V)=0$ for $m\neq 0, l$.*

The proof involves the continuous cohomology and the theory of admissible representations of G. Let me show how these enter into consideration.

First of all, one may assume $\boldsymbol{G}$ to be simply connected (see Lemma 3.4 in [**1**]). Let

$$I_V = \operatorname{Ind}(V \mid \Gamma, G)$$

be the space of all locally constant functions $f\colon G\to V$ such that $f(\gamma g)=\gamma\cdot f(g)$ for all $\gamma\in\Gamma$, $g\in G$. The group G acts on this by right translation: $R_g f(x)=f(xg)$. With this action, since $\Gamma\backslash G$ is compact, the space I_V is that of an admissible representation of G, i.e., every element in this space is fixed by some compact open subgroup of G, and for any compact open subgroup K, the space $I_V^K=\{f\in I_V \mid R(k)f=f \text{ for all } k\in K\}$ has finite dimension. Since V is a unitary Γ-module and there exists a G-invariant measure on $\Gamma\backslash G$, the representation of G on I_V is unitary as well. Admissibility and unitarity together of I_V imply easily that it is G-isomorphic to a direct sum $\bigoplus I_n$ of irreducible, admissible, unitary G-spaces, each isomorphism class occurring with finite multiplicity.

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