BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 5, September 1974

REPRESENTATION OF PARTIALLY ORDERED LINEAR ALGEBRAS

BY RICHARD C. METZLER

Communicated by Robert Bartle, December 12, 1973

In [**2**] and [**3**] a condition on partially ordered linear algebras (*pola*'s) is defined, and it is shown that Dedekind σ-complete polas satisfying this condition have many of the properties of function spaces. Using a theorem of H. Nakano we can show, even without the hypothesis that the pola is Dedekind σ-complete, that any such pola is isomorphic to a pola of continuous, almost-finite, extended-real-valued functions. If A is a pola with multiplicative identity 1 the condition mentioned is:

P_1. If $x \in A$ and $x \geqq 1$, then x has an inverse and $x^{-1} \geqq 0$.

THEOREM. *In order for an Archimedean pola A with identity* 1 *to be isomorphic to a pola of continuous, almost-finite, extended-real-valued functions on a compact Hausdorff space X, it is sufficient that* P_1 *hold for A. The condition is necessary also if $A_1 = \{y \in A$: there exists $\alpha \in R^+$ with $-\alpha 1 \leqq y \leqq \alpha 1\}$ is complete in the order unit norm derived from* 1 *and if the image of A_1 separates points in X.*

PROOF. The standard completion procedure for Archimedean ordered linear spaces shows that A is isomorphic with an order dense subspace $\hat{A}$ of a Dedekind complete linear lattice D. In [**4**, p. 150] it is shown that the multiplication on $\hat{A}$ can be extended to D in such a way that D is a pola if the following continuity condition is satisfied: For every subset B of A, $\inf B = 0$ implies $\inf(aB) = \inf(Ba) = 0$ for all positive elements a in A. Given P_1, multiplication by $(a+1)^{-1}$ shows this condition is satisfied. Thus D is a linear lattice pola and the order density of $\hat{A}$ shows (since 1 is easily seen to be a weak order unit for A) that the image of 1 is a weak order unit for D. Now D (and hence A) has a representation of the type desired by [**1**, Corollary, p. 625].

To prove the second statement we note that the assumptions, together with the Stone–Weierstrass theorem, give the result that if $A \to \hat{A}$ is the isomorphism then $\hat{A}_1 = C(X)$. Then, given any x in A such that $x \geqq 1$,

---

AMS (MOS) subject classifications (1970). Primary 06A70; Secondary 06A65.
Key words and phrases. Partially ordered linear algebra, representation by functions.