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2^X AND $C(X)$ ARE HOMEOMORPHIC TO THE HILBERT CUBE[1]

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1. **Introduction.** Let 2^X be the hyperspace of nonempty closed subsets of a metric continuum X, and let $C(X)$ be the space of nonempty subcontinua of X, both with the Hausdorff metric. This paper is a description of the general techniques used in obtaining the following results.

THEOREM 1. *$2^X \approx Q$, the Hilbert cube, if and only if X is a nondegenerate Peano space (locally connected metric continuum).*

THEOREM 2. *$C(X)\times Q \approx Q$ if and only if X is a Peano space, and $C(X) \approx Q$ if and only if X is a nondegenerate Peano space containing no free arcs.*

Theorem 1 answers a question posed by Wojdyslawski [**8**], who later showed that 2^X is an AR for every Peano space X [**9**]. The converse is easily seen to be true; in fact, if 2^X is locally connected, then so is X. The proof of Theorem 1 is based on the recent result of Schori and West [**5**] that $2^\Gamma \approx Q$ for every nondegenerate connected graph Γ.

Wojdyslawski also showed that $C(X)$ is an AR if (and only if) X is a Peano space. An important special case of Theorem 2 is already known: if Γ is a connected graph, then $C(\Gamma)$ is a contractible polyhedron [**4**], and therefore $C(\Gamma)\times Q \approx Q$ by a theorem of West [**6**]. Since $C(I) \approx I^2$, the condition that X contains no free arcs is clearly necessary for $C(X) \approx Q$. The proof of sufficiency uses a recent result of West [**7**] that $C(D) \approx Q$ for every dendron D with a dense set of branch points.

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