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QUADRATIC SPLINE INTERPOLATION[1]

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ABSTRACT. A quadratic spline interpolation theory is developed which, in general, produces better fits to continuous functions than does the existing cubic spline interpolation theory.

1. Let $\Delta:0=x_0<x_1<\cdots<x_n=1$ be a partition of $[0, 1]$. A function s is a spline of order m having knots in Δ if $s \in C^{m-2}[0, 1]$ and, on each interval (x_{i-1}, x_i), $s(x)$ is represented by a polynomial of degree $m-1$ or less.

For the case $m=3$, we call s a quadratic spline. For quadratic splines, set $s_i=s(x_i)$, $\lambda_i=s'(x_i)$ for $i=0, 1, \cdots, n$, and $h_i=x_i-x_{i-1}$, $s_{i-1/2}=s(x_i-h_i/2)$, $a_i=h_{i+1}/(h_i+h_{i+1})$, $c_i=1-a_i$ for $i=1, 2, \cdots, n$.

Any three of the parameters s_{i-1}, $s_{i-1/2}$, s_i, λ_{i-1}, λ_i may be used to represent the quadratic spline s on the interval (x_{i-1}, x_i). Because of continuity, these parameters must satisfy the consistency relations

$$a_i s_{i-1} + 3s_i + c_i s_{i+1} = 4a_i s_{i-1/2} + 4c_i s_{i+1/2} \tag{1.1}$$

and

$$c_i\lambda_{i-1} + 3\lambda_i + a_i\lambda_{i+1} = 8(s_{i+1/2} - s_{i-1/2})/(h_i + h_{i+1}) \tag{1.2}$$

for $i=1, 2, \cdots, n-1$. For simplicity, we assume that s and s' are periodic, i.e.

$$s_0 = s_n \quad \text{and} \quad \lambda_0 = \lambda_n \tag{1.3}$$

so that (1.1) and (1.2) hold for $i=0$ and $i=n$ provided that the subscripts be read modulo n. For a given Δ, the periodic quadratic spline subspace has dimension n.

2. If y is a given continuous function satisfying

$$y(0) = y(1), \tag{2.1}$$

the periodic quadratic spline interpolant $s=S_3y$ associated with y and Δ is

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