HIGHER DIMENSIONAL DIOPHANTINE PROBLEMS

BY SERGE LANG

As on previous occasions [L 5], [L 6], I shall discuss some general conjectures concerning diophantine analysis on varieties. These involve rational points, integral points, and the possibility of treating by diophantine methods questions which in the past have been handled by congruence methods.

1. **Rational points.** A classical conjecture of Mordell states that a curve of genus ≥ 2 over the rational numbers has only a finite number of rational points. Let K be a finitely generated field over the rational numbers. Then the same statement should hold for a curve defined over K, and a specialization argument due to Néron shows in fact that this latter statement is implied by the corresponding statement over number fields (cf. [L 1, Chapter VII, §6]).

Let V be a variety in projective space, defined over the complex numbers, and therefore over some finitely generated field over the rationals. We shall say that V has the **Mordell property** if it has only a finite number of rational points in any finitely generated field over Q. One possibility to extend Mordell's conjecture to higher dimensional varieties is as follows.

(1.1)　*Let D be a bounded domain in C^n (it should be irrelevant whether D is symmetric or not). Let Γ be a discrete group of automorphisms, acting freely on D, and assume that the quotient $\Gamma\backslash D$ is compact, and embedded as a variety V in projective space. Then V has the Mordell property.*

One must assume that Γ operates freely (the isotropy group at each point is the identity), otherwise the quotient may have singularities, whose effect is analogous to decreasing the genus in the case of curves. Similarly, one must assume that the quotient is compact, otherwise one is faced with a situation which may be like that of modular curves which may have a low genus 0 or 1. The Mordell conjecture is a special case of the above, because a curve of genus ≥ 2 is a quotient of the disc. On the other hand, it has always been useful to regard a curve of genus ≥ 2 as a