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METRIZATION AND PARACOMPACTNESS IN TERMS OF REAL FUNCTIONS

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One of the most useful tools developed over the last fifty years in the theory of metrization of topological spaces is the following: "A T_0 space X is separable metrizable if and only if X has the weak topology induced by countably many real-valued functions." This embedding theorem appears in various forms in most text books, and is customarily used in proving the Urysohn metrization theorem. Of course, the Urysohn metrization theorem is itself a corollary to the Nagata-Smirnov theorem.

It is the purpose of this paper to announce a recent result that will serve as an analogue of the above embedding theorem for arbitrary metric spaces, in that the embedding theorem follows as a corollary and the Nagata-Smirnov theorem follows as a consequence.

Call a collection $\{f_\alpha : \alpha \in A\}$ of real-valued functions defined on a topological space *relatively complete* if $\inf\{f_\beta : \beta \in B\}$ and $\sup\{f_\beta : \beta \in B\}$ are continuous functions for each $B \subset A$. Then it is true that

THEOREM 1. *A T_0 space X is metrizable if and only if X has the weak topology induced by a σ-relatively complete collection.*

The proof is accomplished by showing that X is a paracompact Hausdorff space with a development. The paracompactness is an immediate consequence of the following result.

THEOREM 2. *A T_1 space X is paracompact if and only if for each open cover $\mathscr{W}$, there is a σ-relatively complete collection $\bigcup\{f_{n\alpha} : \alpha \in A_n\}$ and a refinement of $\mathscr{W}$ of sets of the form $\bigcap_{i=1}^{m} f_{n_i\alpha_i}^{-1}(U_i)$ where U_i is open in R.*

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