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NONOSCILLATION AND INTEGRAL INEQUALITIES[1]

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1. **Introduction.** In this note we present a systematical approach to nonoscillation norm conditions for real and complex linear differential systems. We show that the infima of the appropriate integral functionals are constants for nonoscillation criteria. Furthermore in the real case these infima are the best possible nonoscillation constants. Applying an iterative method we prove that the minimal solutions exist and satisfy the Euler-Lagrange equations. This in particular implies that certain first order autonomic systems have periodic solutions. Finally we compute these infima for certain norms. Thus we obtain many known and new results.

2. **The variational problems.** We consider linear differential systems of the form

$$x' = A(t)x \tag{1}$$

in some domain D. Here $A(t)=(a_{jk}(t))_1^n$ is an $n\times n$ matrix and $x(t)=(x_1(t),\cdots,x_n(t))$ is an n column vector. There are two different cases: (i) D is an interval $[a, b]$. In that case $A(t)$ is real piecewise continuous on $[a, b]$. (ii) D is a simply connected domain on the complex plane with a boundary Γ. In that case $A(t)$ is a complex valued analytic matrix in D. The system (1) is called nonoscillatory [**1**] (disconjugate [3]) if any nontrivial solution $x(t)=(x_1(t),\cdots,x_n(t))$ of (1) has at least one component $x_j(t)$ which does not vanish at any point of D. Let $\|x\|_1$ and $\|x\|_2$ be a pair of norms defined on R^n. Assume furthermore that each norm is an absolute norm, i.e. $\|(x_1,\cdots,x_n)\|_j=\|(|x_1|,\cdots,|x_n|)\|_j$, $j=1, 2$. Thus these two norms can be naturally extended to C^n. The matrix norm $\|A\|_{1,2}$ is defined by $\sup\|Ax\|_2/\|x\|_1$. Let T be the collection of all piecewise smooth vectors $x(t)$ on the interval $[a, b]$ which does not vanish at any point of this interval. By S_0 we denote the set of all $x(t)\in T$ and that

$$x_j(a)x_j(b) = 0, \qquad j = 1,\cdots,n. \tag{2}$$

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