

# THE EQUICHARACTERISTIC CASE OF SOME HOMOLOGICAL CONJECTURES ON LOCAL RINGS

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**ABSTRACT.** This is an announcement of proofs of the intersection conjecture of Peskine and Szpiro and, hence, also, of M. Auslander's zerodivisor conjecture and of an affirmative answer to Bass' question for any equicharacteristic local ring  $R$ . The key point is that if  $x = x_1, \dots, x_n$  is a system of parameters for such an  $R$  there exists an  $R$ -module  $M$  (not necessarily of finite type) such that  $(x)M \neq M$  and for each  $k$ ,  $0 \leq k < n$ ,  $(x_1, \dots, x_k)M : x_{k+1}R = (x_1, \dots, x_k)M$ , i.e.  $M$  is a sort of non-Noetherian Cohen-Macaulay module of depth  $n$ .

**0. The main results.** "Ring" means commutative, associative ring with identity, and "local ring" means Noetherian ring with a unique maximal ideal. Unless otherwise indicated  $(R, P)$  denotes an  $n$ -dimensional local ring with maximal ideal  $P$ , and  $x = x_1, \dots, x_n$  a system of parameters (henceforth, s.o.p.) for  $R$ . The first main result is

**THEOREM 1.** *Let  $x = x_1, \dots, x_n$  be a system of parameters for an equicharacteristic local ring  $(R, P)$  of dimension  $n$ . Then there exists an  $R$ -module  $M$  (not necessarily of finite type) such that  $(x)M \neq M$  and for each  $k$ ,  $0 \leq k < n$ ,  $(x_1, \dots, x_k)M : x_{k+1}R = (x_1, \dots, x_k)M$ .*

If the conclusion of Theorem 1 holds for  $x$ ,  $M$  we shall say that  $M$  is  *$x$ -regular*.

The proof of this result is sketched in §1, and will appear in [H4]. (A preliminary version of [H4], which contains the proof, is available in the Aarhus University Preprint Series.)

In the following corollaries of Theorem 1 the hypothesis that a ring  $R$  or  $S$  contain a field may be weakened to the hypothesis that each of its local rings, modulo nilpotents, contain a field.

**COROLLARY 1 (INTERSECTION CONJECTURE OF PESKINE AND SZPIRO).** *Let  $(R, P)$  be a local ring which contains a field, let  $M, N$  be  $R$ -modules of finite type, and suppose that  $\text{Supp}(M \otimes N) = \{P\}$ . Then  $\dim N \leq \text{pd}_R M$ .*

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