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ON VITALI-HAHN-SAKS TYPE THEOREMS

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In recent years extensive work has been done on the Vitali-Hahn-Saks theorem and its relatives. Seever **[13]** considered the question of extending the Vitali-Hahn-Saks theorem to the case where the domain is a Boolean algebra which is not necessarily sigma complete. Brooks and Jewett **[2]** established results for a strongly bounded map defined on a Boolean sigma algebra of sets with values in a Banach space. Further generalizations to group-valued set functions have been studied by the Poznán school (see **[5]**, **[6]**, **[7]**, **[8]**, **[9]**, **[11]**, **[12]**). The work of all these authors is generalized herein to the case of strongly bounded maps defined on Boolean algebras with the Seever property and taking values in a Banach space. Some applications other than those considered herein and the final generalization to the group-valued case can be found in **[10]**.

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1. **Notation and definitions.** A Boolean algebra $\mathscr{B}$ has the *property* (I) if and only if for any sequences $\{x_n\}$ and $\{y_n\}$ in $\mathscr{B}$ satisfying $x_n \leqq y_m$ for all n, m, there exists $x \in \mathscr{B}$ such that $x_n \leqq x \leqq y_n$ for all n. This condition is equivalent to the condition: given any sequences $\{a_n\}$ and $\{b_n\}$ in $\mathscr{B}$ satisfying $a_n \wedge a_m = 0$, $b_n \wedge b_m = 0$ for $n \neq m$ and $a_n \wedge b_m = 0$ for all n, m, there exists an element a in $\mathscr{B}$ such that $a \geqq a_n$ and $b_n \wedge a = 0$ for all n.

Unless signified otherwise, $\mathscr{B}$ will be used in this paper to denote a Boolean algebra with the property (I). The symbol X denotes a Banach space and X^* its Banach space dual.

A finitely additive $\mu : \mathscr{B} \to X$ is *bounded* whenever there exists $M > 0$ such that $\|\mu(b)\| \leqq M$ for all $b \in \mathscr{B}$; μ is said to be *strongly bounded* if $\|\mu(e_n)\| \to 0$ as $n \to \infty$ for each disjoint sequence $e_1, \cdots, e_n, \cdots$ of elements in $\mathscr{B}$. A sequence $\mu_n : \mathscr{B} \to X$, $n = 1, 2, \cdots$, is *uniformly strongly bounded* if for each disjoint sequence $\{e_n\} \subset \mathscr{B}$, $\lim_n \sup_k \|\mu_k(e_n)\| = 0$. By grouping it is easy to see that if μ is strongly bounded and $\{e_n\} \subset \mathscr{B}$ is disjoint, then $\sum_{n=1}^{\infty} \mu(e_n)$ is an unconditionally convergent series in X. A map $\mu : \mathscr{B} \to X$ is

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