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MANIFOLDS WITH THE FIXED POINT PROPERTY. I

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1. **Introduction.** Suppose that $f: M \to M$ is a map of the simply connected closed (smooth or PL) manifold M which preserves a given geometric structure. We shall consider the question of when f has a fixed point. (The geometric structure is described by an element ξ in $K_R(M)$, the Grothendieck group of real vector bundles over M. If $\deg f = 1$, then for f to preserve ξ means just that $f^*\xi = \xi$, and the appropriate notion when $\deg f \neq 1$ is given below in §2. Such maps are said to be (ξ, λ)-maps with λ an integer.) Since M is simply connected, one need only compute the Lefschetz number $\mathscr{L}(f)$ of f. Thus there are three natural stages to the solution: the determination of the induced homomorphism $f^*: H^*(M; Z) \to H^*(M; Z)$ first below the middle dimension, then in the middle dimension (when dim M is even), and finally the determination of how the two are related to each other and how they determine the behaviour above the middle dimension.

As a first step in this direction, we consider here the case of $(2m-1)$-connected M of dimension $4m$ whose intersection pairing is definite (said to be of class $\mathscr{M}_{4m}$). It is shown that if ξ is asymmetric enough in a suitable sense (described below in §2), then any (ξ, λ)-map $f: M \to M$ has a fixed point. In particular it follows that if the tangent bundle $\tau(M)$ of M is asymmetric enough, then a $(\tau M, 1)$-map $f: M \to M$ has a fixed point. Therefore every homeomorphism of such a manifold M has a fixed point. It is also shown that the product of (ξ, λ)-maps with ξ being asymmetric also has a fixed point.

[*Note.* At this point I would like to thank Ed Fadell for the suggestions and stimulation offered in many good conversations on this topic.]

2. **Statement of results.** Suppose that M is a smooth (or PL) simply connected closed manifold of dimension $4m$. A map $f: M \to M$ is said to be a (ξ, λ)-*map*, where λ is an integer, if and only if $f^*\xi = \lambda\xi + p^*\eta$ where

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