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SOME NEW RESULTS ABOUT HAMMERSTEIN EQUATIONS[1]

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Let Ω be a σ-finite measure space. Let K be a (nonlinear) montone operator and let $(Fu)(x)=f(x, u(x))$ be a Niemytski operator. We consider the Hammerstein type equation

$$u + KFu = g. \tag{1}$$

A detailed discussion and a complete bibliography about equation (1) can be found in [**3**]. The new feature about the results we present here is the fact that we do not assume any coercivity for F. When F is monotone and K maps $L^1(\Omega)$ into $L^\infty(\Omega)$, there is no growth restriction on F either (cf. Theorem 1). The monotonicity of F can be weakened when K is compact (cf. Theorem 4). Also some of these results are valid for systems in the case where F is the gradient of a convex function (cf. Theorem 5).

Assume

(2) K is a monotone hemicontinuous mapping from $L^1(\Omega)$ into $L^\infty(\Omega)$ which maps bounded sets into bounded sets,

(3) $f(x, r): \Omega \times \boldsymbol{R} \to \boldsymbol{R}$ is continuous and nondecreasing in r for a.e. $x \in \Omega$, and is integrable in x for all $r \in \boldsymbol{R}$.

THEOREM 1. *Under the assumptions* (2) *and* (3), *equation* (1) *has one and only one solution* $u \in L^\infty(\Omega)$ *for every* $g \in L^\infty(\Omega)$.

Uniqueness. Let u_1 and u_2 be two solutions of (1). By the monotonicity of K we get

$$\int_\Omega (u_1(x) - u_2(x)) \cdot (f(x, u_1(x)) - f(x_1, u_2(x)))\, dx \leqq 0$$

which implies that $f(x, u_1(x))=f(x, u_2(x))$ a.e. on Ω and therefore by (1), $u_1=u_2$.

In proving existence of u we shall use the following

LEMMA 1. *Let X be a Banach space and let $K: X \to X^*$ and $F: X^* \to X$ be two monotone hemicontinuous operators. Let $\{u_n\} \subset X^*$, $\{v_n\} \subset X$ and*

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