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HARMONIC QUASICONFORMAL MAPPINGS OF RIEMANNIAN MANIFOLDS

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1. **Introduction.** In this note, we announce some results concerning the distance-volume-decreasing property of harmonic quasiconformal mappings of Riemannian manifolds. Details will appear elsewhere.

Let M and N be C^∞ Riemannian manifolds of dimensions m and n, respectively. Let $f: M \to N$ be a C^∞ mapping. The Riemannian metrics of M and N can be written locally as $ds_M^2 = \omega_1^2 + \cdots + \omega_m^2$ and $ds_N^2 = \omega_1^{*2} + \cdots + \omega_n^{*2}$, where ω_i $(1 \leqq i \leqq m)$ and ω_a^* $(1 \leqq a \leqq n)$ are linear differential forms in M and N, respectively. The structure equations in M are

$$d\omega_i = \sum_j \omega_j \wedge \omega_{ji},$$
$$d\omega_{ij} = \sum_j \omega_{ik} \wedge \omega_{kj} - \frac{1}{2} \sum_{k,l} R_{ijkl} \omega_k \wedge \omega_l.$$

Similar equations are valid in N and we will denote the corresponding quantities in the same notation with asterisks. Let $f^* \omega_a^* = \sum_i A_i^a \omega_i$. Then the covariant differential of A_i^a is defined by

$$DA_i^a \equiv dA_i^a + \sum_j A_j^a \omega_{ji} + \sum_b A_i^b \omega_{ba}^* \equiv \sum_j A_{ij}^a \omega_j$$

with $A_{ij}^a = A_{ji}^a$. The mapping f is called harmonic (resp. totally geodesic) if $\sum_i A_{ii}^a = 0$ (resp. $A_{ij}^a = 0$).

If $m = n$, then at each point of M the matrix (A_i^a) has the adjoint (B_a^i). Let C be the scalar invariant $\sum B_a^i B_b^k A_{kj}^a A_{ij}^b$. In [2], Chern and one of the authors proved the following theorems which may be regarded as extensions of Schwarz's lemma.

THEOREM I. *Let B^n be the n-dimensional open ball with the standard hyperbolic metric and N an n-dimensional Riemannian manifold. Let $f: B^n \to N$ be a harmonic mapping satisfying the condition $C \leqq 0$. If N is an Einstein manifold with scalar curvature $R^* \leqq -4n(n-1)$ or if the sectional curvature of N is $\leqq -4$, then f is volume-decreasing.*

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