BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 3, May 1974

LOCALLY PRIME ARCS WITH FINITE PENETRATION INDEX

BY J. McPHERSON AND GEOFFREY HEMION

Communicated by O. G. Harrold, September 29, 1973

Introduction. Let k be an oriented arc in R^3, which has an isolated wild point p at which the penetration index P of k is finite and no less than three. We say that k is locally prime at p if there is a tame closed 3-cell neighbourhood U of p which meets k on its boundary in exactly P points, such that if Γ is any cube in Int U which meets k on its boundary in two points, then it is impossible for k to be knotted in Γ; that is, $\pi_1(\Gamma - k)$ must be free cyclic. For example, the arc shown in Figure 2 of [2] is not locally prime at its wild endpoint, whereas the arcs $A_1, A_2, \cdots$ of [1] are all locally prime at their respective endpoints.

The purpose of this paper is to announce the existence of a factorisation theorem for arcs which are not locally prime at their isolated wild points, to the effect that each nonlocally-prime arc can be decomposed into the "product" of a locally prime arc with a sequence of tame knots, and that this decomposition is unique; this extends the 1961 result of Fox and Harrold [5]. The proofs, which rely heavily on (sometimes tortuous) cutting and pasting arguments, will appear in another paper.

The second author is presently working on a more general factorisation theorem for arcs with isolated wild points and finite penetration index, to the effect that any such arc may be written as a finite "product" of arcs, so that each term in the product is an arc which is not itself the composite of two other wild arcs.

We have not yet explored the possibility of a more general theorem for arcs which are locally knotted in the sense of [9].

Results. Except in Theorem 1, we assume that our knots and arcs have only one wild point; the generalisation to knots and arcs with finitely many wild points is easily accomplished.

A *cube* is any tame set homeomorphic to I^3; it is *well-placed* (with respect to k) if it meets k on its boundary in exactly two points. If Γ is a well-placed cube, we say that *k represents the knot κ in Γ* if joining the endpoints of $k \cap \Gamma$ by a simple arc on Bd Γ yields an oriented knot equivalent to κ in R^3, and Γ is a *k-prime* cube if k represents a (nontrivial) prime knot in Γ. We use the following fact from [10]:

LEMMA A. *If k represents the knot $\pi_1 \# \cdots \# \pi_m$ in Γ (where each π_i is an oriented tame prime knot, and $\#$ denotes tame knot multiplication),*

AMS (MOS) subject classifications (1970). Primary 55A30.