BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 3, May 1974

MARKET REPRESENTATIONS OF n-PERSON GAMES

BY LOUIS J. BILLERA[1] AND ROBERT E. BIXBY[2]

Communicated by Alex Rosenberg, September 22, 1973

ABSTRACT. The class of n-person games (without side payments) obtainable from a certain economic market model is characterized as the class of totally balanced games. Various market representations for a given totally balanced game are considered, and a finite commodity representation without production is obtained.

1. **Introduction.** It is the purpose of this announcement to describe some results on the problem of characterizing a certain class of n-person games which arise from considerations in economics. The details of these can be found in several different papers [2], [3], [4] appearing elsewhere. In addition, we will briefly discuss an equivalence notion for economic markets, and describe an equivalent form of one of the representations in [2].

We first establish some notation. Let $N=\{1, \cdots, n\}$ and $2^N=\{S\subset N \mid S\neq\varnothing\}$. For $S\in 2^N$, let $R^S=\{x\in R^n \mid x_i=0 \text{ for } i\notin S\}$ and $R^S_+=R^S\cap R^n_+$, where $R^n_+=[0, \infty)^n$. If $A, B\subset R^n$ and $\alpha\in R_+$, we write $A-B=\{a-b \mid a\in A, b\in B\}$ and $\alpha A=\{\alpha a \mid a\in A\}$. For $S\subset N$, let $\chi_S\in R^n$ be given by $(\chi_S)_i=1$ if $i\in S$, $(\chi_S)_i=0$ if $i\notin S$.

DEFINITION 1.1. A *game* V on N is a function from 2^N to subsets of R^n, such that for each $S\in 2^N$, $V(S)=C_S-R^S_+$ where $C_S\subset R^S$ is nonempty, compact and convex.

DEFINITION 1.2. A *market* on N is a set $\{(X^i, Y^i, u_i, \omega^i) \mid i\in N\}$ where for each $i\in N$,

(1.2.1) X^i, Y^i are nonempty, compact, convex subsets of a real Hausdorff linear topological space E,

(1.2.2) $u_i: X^i\to R$ is concave and upper-semicontinuous, and

(1.2.3) $\omega^i\in X^i-Y^i$.

A market is a simple economic model of a production-trading situation involving n participants, called traders. The space E corresponds to the space of economic goods. The sets X^i and Y^i, called consumption and

AMS (MOS) subject classifications (1970). Primary 90D12, 90A15.

Key words and phrases. Market game, game without side payments, economic market, totally balanced game, Pareto surface, utility function, trading market, production market.

[1] Partially supported by the National Science Foundation under grant GP 32314X and by the Office of Naval Research under Contract N00014-67-A-0077-0014.

[2] Partially supported by the Office of Naval Research under contract N00014-67-A-0077-0014 at Cornell University.