

## OPEN SETS OF POINTS WITH GOOD STABILIZERS

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**1. Observable groups and invariant theory [2].** Throughout this paper,  $k$  denotes a fixed algebraically closed field of characteristic 0 and  $G$  a connected, reductive algebraic group defined over  $k$ . (Our definitions and theorems regarding algebraic groups are taken from [1]. From now on, we shall always write variety instead of  $k$ -variety and morphism instead of  $k$ -morphism. A variety will always be identified with its  $k$ -rational points.) The subgroups of  $G$  in which we are interested are described in the next two definitions.

An algebraic subgroup  $H$  of  $G$  is called *observable in  $G$*  if there is a finite-dimensional rational representation  $\rho: G \rightarrow GL(V)$  and a vector  $v \in V$  such that  $H = S_G(\rho, v) = \{g \in G \mid \rho(g)v = v\}$ .

An observable subgroup  $H$  of  $G$  is said to satisfy the *codimension 2 condition on  $G/H$*  if there is a finite-dimensional rational representation  $\rho: G \rightarrow GL(V)$  and a vector  $v \in V$  such that

(a)  $H = S_G(\rho, v)$ ;

(b) each irreducible component of  $\text{Cl}(G \cdot v) - G \cdot v$  has codimension  $\geq 2$  in  $\text{Cl}(G \cdot v)$ .

(In (b),  $G \cdot v$  denotes the orbit of  $v$  under the action of  $G$ . Furthermore, if  $A$  is a subset of some affine space  $k^m$ , we shall always denote by  $\text{Cl}(A)$  the Zariski-closure of  $A$  in  $k^m$ .)

Now let  $Z$  be an irreducible affine variety and let  $k[Z]$  be the ring of regular functions on  $Z$ . We assume that  $G$  operates regularly on  $Z$  via a mapping from  $G \times Z \rightarrow Z$  denoted by  $(g, z) \rightarrow g \cdot z$ . Then  $G$  operates on  $k[Z]$  as follows:  $(g \cdot f)(z) = f(g^{-1} \cdot z)$  for all  $f \in k[Z]$ ,  $z \in Z$ , and  $g \in G$ . For  $H$  any subgroup of  $G$  we put  $k[Z]^H = \{f \in k[Z] \mid h \cdot f = f \text{ for all } h \in H\}$ .

The importance of observable groups in invariant theory is illustrated by the two following results.

(i) Given any subgroup  $H$  of  $G$ , there is an observable subgroup  $H''$  of  $G$  such that for any irreducible affine variety  $Z$  on which  $G$  operates regularly, we have  $k[Z]^H = k[Z]^{H''}$ .

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