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SMOOTH MAPS OF CONSTANT RANK

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1. **Introduction.** In this announcement the Smale-Hirsch classification of immersions ([8], [5]) is extended to maps of arbitrary constant rank, under certain conditions on the source manifold.

THEOREM 1. *If M is open and has a proper Morse function with no critical points of index $>k$, then the differential map $d:\mathrm{Hom}_k(M, W)\to \mathrm{Lin}_k(TM, TW)$ is a weak homotopy equivalence.*

(A manifold with such a Morse function will be said to have *geometric dimension* $\leqq k$. We will write geo dim $M\leqq K$.)

Notation. M and W are smooth manifolds with tangent bundles TM, TW; $\mathrm{Hom}_k(M, W)$ is the space of smooth maps of rank k from M to W, with the C^1-compact-open topology; $\mathrm{Lin}_k(TM, TW)$ is the space of continuous maps: $TM\to TW$ which are fiberwise linear maps of rank k, with the compact open topology; $d(f)=df$.

REMARKS. 1. Weakening the hypotheses leads to false statements. If M is not open there are counterexamples when $k=\dim W$ as in [7]. Otherwise, take M to be the parallelizable manifold $S^{k+1}\times R$; then the identity map of M can be covered by $H\in \mathrm{Lin}_k(TM, TM)$ but H cannot be homotopic to the differential of an $f\in \mathrm{Hom}_k(M, M)$ since such an f (by Sard's theorem) would be null-homotopic. I owe this example to David Frank.

2. When $k=\dim M$ this gives the Smale-Hirsch theorem for open manifolds, but when $k=\dim W$ this does not give the full classification of submersions [7]. The missing cases will be considered in a future article. (ADDED IN PROOF. A necessary and sufficient condition for $H\in \mathrm{Lin}_k(TM, TW)$ to be homotopic to the differential of some $f\in \mathrm{Hom}_k(M, W)$ is given, for arbitrary open M, in M. L. Gromov, *Singular smooth maps*, Mat. Zametki **14** (1973), 509–516. It is equivalent to requiring that H factor through a k-dimensional bundle over a k-dimensional complex.) Immersions and submersions are the only overlap between this theorem and Feit's classification of k-mersions (maps of rank everywhere$\geqq k$) [2]

3. This theorem is not a special case of Gromov's theorem [3], since

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