BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 3, March 1974

NORMAL FIELD EXTENSIONS K/k AND K/k-BIALGEBRAS[1]

BY DAVID J. WINTER

Communicated by George Seligman, August 28, 1973

Throughout the paper K/k is a field extension and p is the exponent characteristic.

In this paper I introduce the notion of K/k-bialgebra (coalgebra over K and algebra over k) and describe a theory of finite dimensional normal field extensions K/k based on a K-measuring K/k-bialgebra $H(K/k)$ (see 1.2, 1.6 and 1.10). This approach to studying K/k was inspired by my conviction that a successful theory would, in view of the Jacobson-Bourbaki correspondence theorem, result from suitably equipping the endomorphism ring $\mathrm{End}_k K$ of K/k with additional structure which would effectively reflect the multiplicative structure of K.

Some initial parts of the theory developed here are parallel to Moss Sweedler's very effective theory of normal extensions based on a universal cosplit K-measuring k-bialgebra (coalgebra over k and algebra over k) [**1**].

In §1 the structure of K/k is related to that of $H(K/k)$. At the same time, general properties of K/k-bialgebras are described. In §2, K-measuring k-bialgebras and semilinear K-measuring K/k-bialgebras are related, and the structure of semilinear conormal K-measuring K/k-bialgebras is described. In §3 the structure of a finite dimensional radical extension K/k and that of its K/k-bialgebra $H(K/k)$ are described in detail in terms of the toral k-subbialgebra T of $H(K/k)$. As an application of the theory of toral subbialgebras, a generalization of a theorem of Jacobson on finite dimensional Lie algebras of derivations of a field K is given in §4.

The material outlined in this paper is the outgrowth of preliminary research described at the 1971 Ohio State University Conference on Lie Algebras and Related Topics. A complete development of this material is given in a forthcoming book [**2**].

AMS (*MOS*) *subject classifications* (1970). Primary 12F05, 12F10, 12F15, 15A78.

Key words and phrases. Field extension, Galois, normal, purely inseparable, bialgebra.

[1] The research described here was supported in part by the National Science Foundation.