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VALUES IN DIFFERENTIAL GAMES[1]

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ABSTRACT. This is an announcement of results to be proved in detail in a later report in book form.

The author introduces a new value, the Ω-value, into differential games, and proves that six well-known values for games of perfect information now in the literature are special cases of it. He proves that two (in the present slightly altered formulation four) values introduced in 1972 by Elliott and Kalton using relaxed controls are equal to the mixed-strategy value introduced by Fleming, in 1964. Thus, of the twelve values just mentioned, there are only two which are essentially distinct: the new Ω-value, and Fleming's 1964 value.

Hamilton-Jacobi equations for the Ω-value and for the Fleming mixed-strategy value are announced as well.

1. **Introduction.** We characterize a *differential game of unit duration of purely terminal type* by the following data:

(i) U and V, compact topological spaces;

(ii) $\mathfrak{f}(x, t, u, v)$, a uniformly bounded vector-valued function continuous on $R^p \times [0, 1] \times U \times V$ and uniformly Lipschitzian in x and t;

(iii) $\varphi(x)$, a uniformly bounded and uniformly Lipschitzian function defined throughout R^p.

There are several methods already in the literature for attaching a value to this differential game. We begin by proposing a new one.

2. **The Ω-value.** Let $0 \leqq \sigma \leqq 1$. We conceive of σ as the proportionate reaction time for the maximizing player. Put $t_n = n/N$, and $t_n^\sigma = t_n + (1-\sigma)/N$, $n = 0, \cdots, N-1$. At time t_0 the maximizer chooses $u_0 \in U$. A predecessor v_{-1} is supposed given *a priori* at that time. Then, at time t_0^σ, the minimizer chooses $v_0 \in V$, and so on. Suppose that at time t_n the *position vector*, having started at time t_0 at a *starting point* x_0, reaches x_n. The maximizer now chooses u_n, and the position vector, starting at $\mathfrak{x}(t_n) = x_n$, follows the differential equation

$$\dot{\mathfrak{x}}(t) = \mathfrak{f}(\mathfrak{x}(t), t, u_n, v_{n-1}) \tag{1}$$

on $[t_n, t_n^\sigma)$, reaching $x_n^\sigma = \mathfrak{x}(t_n^\sigma)$ at time t_n^σ. The minimizer now chooses

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