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GENERALIZED SUPER-PARABOLIC FUNCTIONS

BY NEIL EKLUND

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The purpose of this note is to announce results which generalize potential theory (superharmonic functions) to a broad class of parabolic operators. Many of the properties of superharmonic functions carry over to functions in this new class. Let $Q=\Omega\times(0, T)$ where $\Omega\subset E^n$ is a bounded domain and $T>0$ is a scalar. All functions will be defined on $\bar{Q}$ and will be written as functions of (x, t) with $x\in\bar{\Omega}$ and $t\in[0, T]$.

For $(x, t)\in\bar{Q}$ assume

(a) $a_{ij}(x, t)$ is a bounded, measurable function for $i, j=1, 2, \cdots, n$ and assume there is a constant $\lambda>0$ such that $\sum a_{ij}(x, t)z_iz_j\geqq\lambda|z|^2$ for all $z\in E^n$ and almost all $(x, t)\in Q$.

(b) $c(x, t)\in L^q[0, T; L^p(\Omega)]$ for $n/2p+1/q<1$, $1<p, q\leqq\infty$.

(c) $b_j(x, t), d_j(x, t)\in L^q[0, T; L^p(\Omega)]$ for $j=1, \cdots, n$ and $n/2p+1/q<\frac{1}{2}$, $2<p, q\leqq\infty$.

The parabolic operator under consideration is defined by

$$Lu = u_t - \{a_{ij}(x, t)u_{,i} + d_j(x, t)u\}_{,j} - b_j(x, t)u_{,j} - c(x, t)u$$

where $u_{,j}=\partial u/\partial x_j$ and an index i or j is summed over $1\leqq i, j\leqq n$ whenever it is repeated in a product.

DEFINITION 1. $u(x, t)$ is a *weak solution of* $Lu=0$ in Q if u is locally in $L^2[0, T; H^{1,2}(\Omega)]$ and $\iint_Q [a_{ij}u_{,j}\phi_{,j}+d_j\phi_{,j}u-b_ju_{,j}\phi-cu\phi-u\phi_t]\,dx\,dt=0$ for all $\phi\in C_0^1(Q)$.

Let $\partial_pQ=\{\partial\Omega\times[0, T]\}\cup\{\Omega\times(0)\}$ denote the parabolic boundary of Q. Due to the number of definitions and results, they are stated below with no proofs.

THEOREM 1. *Let $f\in C(\partial_pQ)$ and let $u=u(x, t)$ be the weak solution of the boundary value problem*

$$Lu = 0 \quad on\ Q. \qquad u=f \quad on\ \partial_pQ.$$

Then, to each $(x, t)\in Q$, there corresponds a nonnegative Borel measure

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