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FOLIATIONS AND GROUPS OF DIFFEOMORPHISMS

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John Mather has described a close relation between framed codimension-one Haefliger structures (these form a class of singular foliations), and the group of compactly supported diffeomorphisms of $\boldsymbol{R}^1$, with discrete topology **[11]**, **[12]**, **[14]**. In this announcement I will describe generalizations of his ideas to higher codimension Haefliger structures and groups of diffeomorphisms of arbitrary manifolds. See Haefliger **[7]** for a development of Haefliger structures and their classifying spaces.

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Let $\mathrm{Diff}^r(M^p)$ denote the group of C^r diffeomorphisms of M^p, a closed manifold. Let $\mathrm{Diff}_0^r(M^p)$ denote the connected component of the identity.

THEOREM 1. $\mathrm{Diff}_0^\infty(M^p)$ *is a simple group.*

The proof makes use of both the theorem of Epstein **[4]** that the commutator subgroup of $\mathrm{Diff}_0(M^p)$ is simple, and of the result of M. Herman **[9]** which gives the case M^p is a p-torus.

THEOREM 2. $B\bar{\Gamma}_p^\infty$ *is* $(p+1)$*-connected, where* $B\bar{\Gamma}_p^\infty$ *is the classifying space for framed, codimension* p, C^∞, *Haefliger structures.*

The more usual notation is $F\Gamma_p^\infty = B\bar{\Gamma}_p^\infty$. Haefliger proved **[6]** that $B\bar{\Gamma}_p^r$ is p-connected for $1 \leqq r \leqq \infty$; Mather proved that $B\bar{\Gamma}_1^\infty$ is 2-connected. Theorem 2 means that two C^∞ foliations of a manifold coming from nonsingular vector fields are homotopic as Haefliger structures if and only if the normal bundles are isomorphic.

Theorems 1 and 2 are proven by showing they are related; cf. Theorem 4 for a statement of a relationship.

COROLLARY. $P_1^{[p/2]}$ *is nontrivial in* $H^*(B\Gamma_p^\infty; \boldsymbol{R})$ *where* P_1 *is the first real Pontrjagin class of the normal bundle to the canonical Haefliger structure.*

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