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HOLOMORPHY OF COMPOSITION

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1. **Introduction.** We wish to consider the following two problems for E, F, G Banach spaces over the complex field $\boldsymbol{C}$ and $\mathscr{H}(E; F)$, $\mathscr{H}(F; G)$, $\mathscr{H}(E; G)$ the corresponding spaces of holomorphic functions between them (we follow the definitions and notation given in [**3**]): (1) For what vector subspaces $X \subset \mathscr{H}(E; F)$, $Y \subset \mathscr{H}(F; G)$, $Z \subset \mathscr{H}(E; G)$ and corresponding locally convex topologies τ_X, τ_Y, τ_Z will the composition $\phi: (f, g) \in (X, \tau_X) \times (Y, \tau_Y) \to g \circ f \in (Z, \tau_Z)$ be holomorphic? (2) Investigate the holomorphy of $\phi: \mathscr{H}(U; V) \times \mathscr{H}(V; W) \to \mathscr{H}(U; W)$ for $U \subset E$, $V \subset F$, $W \subset G$ open. We are driven to consider general locally convex topologies on X, Y, Z since if ϕ holomorphic means it is separately continuous, then, in particular, the evaluation $f \in (\mathscr{H}(F; \boldsymbol{C}), \tau) \mapsto f(x) \in \boldsymbol{C}$ is continuous. But from [**1**] and [**2**], if F is, for example, a separable or reflexive infinite dimensional Banach space, then τ is not first countable.

2. **Definitions of holomorphy** [**4**]. Let X and Y be complex locally convex spaces (LCS), and W an open, nonempty subset of X. Then $f: W \to Y$ is said to be *holomorphic* if for every $\xi \in W$ there is a sequence $P_m \in \mathscr{P}(^mX; Y)$ (the space of continuous m-homogeneous polynomials from X to Y), $m=0, 1, \cdots$, such that for each continuous seminorm β on Y, one can find a neighborhood V of ξ in W for which

$$\lim_{M \to \infty} \beta\left[f(x) - \sum_{m=0}^{M} P_m(x - \xi)\right] = 0$$

uniformly for $x \in V$. f is said to be *G-holomorphic* (provided X is Hausdorff) if for each $\xi \in W$, $x \in X$, the map $\lambda \in V \mapsto f(\xi + \lambda x) \in Y$ is holomorphic, where $V = \{\lambda \in \boldsymbol{C}: \xi + \lambda x \in W\}$. We denote the space of holomorphic (G-holomorphic) maps by $\mathscr{H}(W; Y)$ ($\mathscr{H}_G(W; Y)$). f is said to be *amply bounded* if for each continuous seminorm β on Y, $\beta \circ f$ is locally bounded.

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