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FIXED POINTS OF ENDOMORPHISMS OF COMPACT GROUPS

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1. **Introduction.** Let G be a compact, connected Lie group and denote its real Čech cohomology by $H^*(G)$. Then $H^*(G)$ is an exterior algebra with generators $1=z_0, z_1, z_2, \cdots, z_\lambda$; where, by a theorem of Hopf **[3]**, λ is equal to the rank of G (the dimension of a maximal torus). This paper announces some improvements of Hopf's result. The details will be published elsewhere.

2. **Fixed point groups.** For a set X and a function $f: X \to X$, let $\Phi(f)$ denote the set of fixed points of f: those $x \in X$ for which $f(x)=x$. If X is a topological group and f is a homomorphism, we will use the symbol $\Phi_0(f)$ for the component of the group $\Phi(f)$ which contains the identity element of X.

We consider a compact, connected Lie group G and let h be an automorphism of G. Choose algebra generators $1=z_0, z_1, z_2, \cdots, z_\lambda$ for $H^*(G)$ and let $\boldsymbol{H}^*(G)$ denote the linear span of $z_1, z_2, \cdots, z_\lambda$. The automorphism h^* of $H^*(G)$ induced by h takes $\boldsymbol{H}^*(G)$ to itself; let $\boldsymbol{h}^*$ denote the restriction of h^* to $\boldsymbol{H}^*(G)$.

Our main result is

THEOREM 1. *Let G be a compact, connected Lie group and let h be an automorphism of G. Then the rank of the Lie group $\Phi_0(h)$ is equal to the dimension of the vector space $\Phi(\boldsymbol{h}^*)$.*

Note that Theorem 1 reduces to Hopf's theorem when h is the identity function.

One might suspect that Theorem 1 is a consequence of some more intimate relationship between $H^*(\Phi_0(h))$ and $\Phi(h^*)$. However, let $g \in G$ be a regular element and define $h(x)=g^{-1}xg$, for $x \in G$, then h induces the identity isomorphism in cohomology, so $\Phi(h^*)=H^*(G)$; while $\Phi_0(h)$ is a maximal torus of G. Thus the possibilities for such a relationship are very limited.

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