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ACTIONS OF REDUCTIVE GROUPS ON REGULAR RINGS AND COHEN-MACAULAY RINGS

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0. **The main results.** This note is an announcement of the results below, whose proofs will appear separately [7].

MAIN THEOREM. *Let G be a linearly reductive affine linear algebraic group over a field K of arbitrary characteristic acting K-rationally on a regular Noetherian K-algebra S. Then the ring of invariants $R=S^G$ is Cohen-Macaulay.*

THEOREM. *If S is a regular Noetherian ring of prime characteristic $p>0$, and R is a pure subring of S (i.e. for every R-module M, $M \to M \otimes_R S$ is injective), e.g. if R is a direct summand of S as R-modules, then R is Cohen-Macaulay.*

The proofs utilize results of interest in their own right:

PROPOSITION A. *Let L be a field, $y_0, \cdots, y_m$ indeterminates over L, $S=L[y_0, \cdots, y_m]$, and $Y=\operatorname{Proj}(S)=\boldsymbol{P}_L^m$. Let K be a subfield of L, and let R be a finitely generated graded K-algebra with $R_0=K$. Let $h: R \to S$ be a K-homomorphism which multiplies degrees by d. Let P be the irrelevant maximal ideal of R, and let $X=\operatorname{Proj}(R)$. Let $U=Y-V(h(P)S)$. Let $\varphi=h^*$ be the induced K-morphism from the quasi-projective L-variety U to the projective K-scheme X. Then $\varphi_i^*: H^i(X, \mathcal{O}_X) \to H^i(U, \mathcal{O}_U)$ is zero for $i \geqq 1$.*

PROPOSITION A′. *Let (R, P) be a local ring of prime characteristic $p>0$ and let h be a homomorphism of R into a regular Noetherian domain S. Suppose that for a certain i the local cohomology module $H_P^i(R)$ has finite length. Then if $i \neq 0$ or $h(P) \neq 0$, the induced homomorphism $H_P^i(R) \to H_{PS}^i(S)$ is zero.*

1. **Applications and corollaries.** We note that the Main Theorem is stronger than the prior conjectures [**2**, §0] or [**3**, p. 56], where S was

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