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INTERPOLATION OF OPERATORS FOR Λ SPACES

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Lorentz and Shimogaki [2] have characterized those pairs of Lorentz Λ spaces which satisfy the interpolation property with respect to two other pairs of Λ spaces. Their proof is long and technical and does not easily admit to generalization. In this paper we present a short proof of this result whose spirit may be traced to Lemma 4.3 of [4] or perhaps more accurately to the theorem of Marcinkiewicz [5, p. 112]. The proof involves only elementary properties of these spaces and does allow for generalization to interpolation for n pairs and for M spaces, but these topics will be reported on elsewhere.

The Banach space Λ_ϕ [1, p. 65] is the space of all Lebesgue measurable functions f on the interval $(0, l)$ for which the norm

$$\|f\|_\phi = \int_0^l f^*(s)\phi(s)\, ds$$

is finite, where ϕ is an integrable, positive, decreasing function on $(0, l)$ and f^* (the decreasing rearrangement of $|f|$) is the almost-everywhere unique, positive, decreasing function which is equimeasurable with $|f|$.

A pair of spaces $(\Lambda_\phi, \Lambda_\psi)$ is called an interpolation pair for the two pairs $(\Lambda_{\phi_1}, \Lambda_{\psi_1})$ and $(\Lambda_{\phi_2}, \Lambda_{\psi_2})$ if each linear operator which is bounded from Λ_{ϕ_i} to Λ_{ψ_i} (both $i=1, 2$) has a unique extension to a bounded operator from Λ_ϕ to Λ_ψ.

THEOREM (LORENTZ-SHIMOGAKI). *A necessary and sufficient condition that $(\Lambda_\phi, \Lambda_\psi)$ be an interpolation pair for $(\Lambda_{\phi_1}, \Lambda_{\psi_1})$ and $(\Lambda_{\phi_2}, \Lambda_{\psi_2})$ is that there exist a constant A independent of s and t so that*

$$(*) \qquad \Psi(t)/\Phi(s) \leqq A \max_{i=1,2}(\Psi_i(t)/\Phi_i(s))$$

holds, where $\Phi(s)=\int_0^s \phi(r)\, dr, \cdots, \psi_2(t)=\int_0^t \Psi_2(r)\, dr$.

PROOF. We only sketch the proof of the necessity since it is standard.

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