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STABLE HOMOTOPY THEORY OVER A FIXED BASE SPACE

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A spectral sequence which may be regarded as an 'Adams spectral sequence over a fixed space B' has been constructed by J. F. McClendon [**6**] and J.-P. Meyer [**8**]. This note describes a generalization in which there is no need for any orientability assumptions. The construction is carried out in a suitable stable category, which may be of independent interest. An application to the enumeration of immersions is given. Details will appear elsewhere.

1. **A stable category.** Let $Ex\text{-}B$ denote the category of ex-spaces (in the terminology of [**4**]) of the path-connected complex B. It is known that the corresponding homotopy category $(Ex\text{-}B)_h$ exhibits certain stability properties [**4**, Theorem 6.4]. One can construct a category $\mathscr{S}/B$, in which the corresponding stable homotopy theory can be investigated, by formalizing the notion of a 'bundle' over B with fibre a CW spectrum (in the sense of [**3**], [**11**]). The details are as follows. If F_1, F_2, F are objects of the category $\mathscr{S}$ of CW spectra, there is a simplicial set of morphisms $\mathrm{Mor}_{\mathscr{S}}(F_1, F_2)$; and $\mathrm{Mor}_{\mathscr{S}}(F, F)$ is a simplicial monoid whose invertible elements form a simplicial group $\mathrm{Aut}_{\mathscr{S}}F$. We take B to be a simplicial set rather than a space.

DEFINITION. An *object* of $\mathscr{S}/B$ is a pair (F, ξ) where $F \in \mathrm{ob}\,\mathscr{S}$ and ξ is a principal simplicial $\mathrm{Aut}_{\mathscr{S}}\,F$-bundle over B.

A *morphism* from (F_1, ξ_1) to (F_2, ξ_2) is a section of the simplicial bundle with fibre $\mathrm{Mor}_{\mathscr{S}}(F_1, F_2)$ associated to the principal $(\mathrm{Aut}_{\mathscr{S}}F_1 \times \mathrm{Aut}_{\mathscr{S}}F_2)$-bundle $\xi_1 \times \xi_2$ over B.

This category inherits much of the usual machinery of stable homotopy theory from Boardman's category $\mathscr{S}$; for example, it has an invertible translation-suspension functor S_B, arbitrary wedges, and smash-product functors. The corresponding homotopy category $(\mathscr{S}/B)_h$ is additive, and triangulated with respect to S_B, and the axioms of Puppe [**9**] hold. There is a *stabilization functor* $(Ex\text{-}B)_h \to (\mathscr{S}/B)_h$ which is bijective on morphism-sets $[X, Y]_B$ of $(Ex\text{-}B)_h$ whenever X is a relative CW ex-space of B, Y is an ex-space fibred over B (with fibre F, say) and $\dim(X-B) \leqq 2\,\mathrm{conn}\,F$.

2. **A cohomology theory on $\mathscr{S}/B$.** Let p be a prime, and let $\rho: \pi_1 B \to GL(V)$ be a semisimple representation of $\pi_1 B$ on a finite-dimensional vector

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