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RESTRICTED IDEALS IN RINGS OF ANALYTIC FUNCTIONS

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Introduction. Let Y be a connected, noncompact Riemann surface, and let A be the ring of all analytic functions on Y. It is known that the ideal theory of the ring A is strikingly similar to the ideal theory of the ring $C(X)$ of all real valued continuous functions on a completely regular topological space X. We show that locally much of the ideal theory of A can be recovered from the ideal theory of $C(\Sigma)$ for a particular space Σ. This will provide a device for transforming results about the ideal theory of $C(\Sigma)$ into results about the ideal theory of A.

1. Let M be a free maximal ideal of A, and let P^* denote the ideal $\bigcap_{n\in N} M^n$. P^* is the largest prime ideal properly contained in M. Let A_{P^*} be the localization of A at P^*. We show in this section that the ideal theory of A_{P^*} is essentially the same as the ideal theory of $C(\Sigma)/P$ for a suitably chosen space Σ and a suitably chosen minimal prime ideal P of $C(\Sigma)$. Let $t \in M-\{0\}$. $Z(t)$, the set of zeros of t, is a countably infinite closed discrete subset of Y. Denote $Z(t)$ by N; we think of $Z(t)$ as a copy of the space N of positive integers. The collection

$$\mu=\{Z(f)\cap N: f\in M\}$$

is a free ultrafilter on N and hence corresponds to a point σ of $\beta N-N$. Let Σ be the space $N\cup\{\sigma\}$, where Σ has the relative topology of βN, and let P be the minimal prime ideal of $C(\Sigma)$ given by

$$P = \{f\in C(\Sigma): Z(f) \cap N \in \mu\}.$$

The ideals of A_{P^*} (respectively $C(\Sigma)/P$) under multiplication of ideals and inclusion form an ordered semigroup $\mathscr{I}(A_{P^*})$ (respectively $\mathscr{I}(C(\Sigma)/P)$).

PROPOSITION 1. *There exists an order preserving isomorphism of* $\mathscr{I}(A_{P^*})$ *onto* $\mathscr{I}(C(\Sigma)/P)$ *that maps the set of principal ideals of* A_{P^*} *onto the set of principal ideals of* $C(\Sigma)/P$.

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