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A FORMULA FOR THE SOLUTION OF THE NAVIER-STOKES EQUATIONS BASED ON A METHOD OF CHORIN

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1. **Introduction.** Recently A. Chorin has found a numerical scheme for solving the Navier-Stokes equations which has the pleasing feature of not breaking down at high Reynolds numbers R [**5**]. The purpose of this announcement is to present a formula ((2) below) which is designed to establish the convergence of Chorin's time step iteration procedure, assuming that the relevant equations (heat equation and Euler's equations) are solved exactly at each step. Computing the error in the steps involving Euler's equation has been studied by Dushane [**6**].

For the case of regions with no boundary, the formula has been established by Ebin-Marsden [**7**] and Marsden [**9**]. That case is not of direct physical interest since it is shown that as $R \to \infty$, the solutions converge to the solutions of the Euler equations. This is interpreted as showing a lack of turbulence.

For regions with boundary, where we expect turbulent phenomena, the formula has an interesting new feature due to the lack of compatibility of boundary conditions of the Euler and Navier-Stokes equations. The new term in the formula has the effect of creating vorticity at the boundary.

What we aim to show is that the formula yields, in the limit as the size of the time step $\to 0$, an *exact* solution of the Navier-Stokes equations. The formula can actually be used as an existence theorem, and for fixed initial data, yields a time of existence independent of R as $R \to \infty$. This is a fundamental improvement over known existence theorems for these equations [**8**]. *This should imply the existence of smooth turbulent solutions of the Navier-Stokes equations* (see [**8**], [**11**]). Moreover, the formula converges as long in time as there is known to be an a priori smooth solution of the Navier-Stokes equations.

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