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A DYNAMICAL SYSTEM ON E^4 NEITHER ISOMORPHIC NOR EQUIVALENT TO A DIFFERENTIAL SYSTEM

BY W. C. CHEWNING

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ABSTRACT. We note that a certain dynamical system on E^4 has local sections which are not classical 3-manifolds. This dynamical system cannot be isomorphic or geometrically equivalent to a differential system on E^4.

Problems 8 and 9 of [3, p. 225] raise the question whether each dynamical system defined on a differentiable manifold is isomorphic or topologically equivalent to a differential system. The purpose of this note is to supply a dynamical system on E^4 which gives a negative answer to the above questions.

DEFINITIONS. A dynamical system on a topological space X is a triple (X, E, π) where E=real number line and $\pi: X\times E\to X$ is a continuous map with the properties that for each $x \in X$, $t_1, t_2 \in E$, $\pi(x, 0)=x$ and $\pi(\pi(x, t_1), t_2)=\pi(x, t_1+t_2)$. A trajectory of (X, E, π) is a set $\pi(\{x\}\times E)$ for a fixed $x \in X$. A rest point of (X, E, π) is a point in X which is also a trajectory.

A local section of extent $\varepsilon>0$ for (X, E, π) is a subset $S\subset X$ with the property that the restriction of π to $S\times(-\varepsilon, \varepsilon)$ is a topological embedding into X. S generates neighborhoods for $K\subset X$ if, for every $\delta>0$, K is interior to $\pi(S\times(-\delta, \delta))$. If S is a local section of extent $\varepsilon>0$, we write $S\pi(-\delta, \delta)$ for $\pi(S\times(-\delta, \delta))$, $0<\delta<\varepsilon$.

THEOREM 1. *Let X be a T_2 topological space, and (X, E, π) a dynamical system on X. Suppose that S and T are each locally compact local sections of extent $\varepsilon>0$ which generate neighborhoods for a point $p \in X$. Then there are relatively open subsets $U\subset S$, $V\subset T$, each containing p, with U homeomorphic to V.*

PROOF. For any space Y, let P_R denote the projection mapping of $Y\times(-\varepsilon, \varepsilon)$ onto $(-\varepsilon, \varepsilon)$. Because $\pi: S\times(-\varepsilon, \varepsilon)\to S\pi(-\varepsilon, \varepsilon)$ is a homeomorphism, $s(x)\equiv P_R\circ\pi^{-1}(x)$ is a continuous map from $S\pi(-\varepsilon, \varepsilon)$ to

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