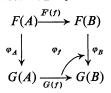
## **QUASI-KAN EXTENSIONS FOR 2-CATEGORIES**

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1. Introduction. Let Cat denote the category of small categories and functors. Cat is a Cartesian closed category, [2] and the prefix 2will denote categories and functors enriched in Cat. 2-Cat denotes the category of small 2-categories and 2-functors. It is also Cartesian closed, but there is another notion of a transformation between 2-functors F and G which has interesting properties; namely a quasi-natural transformation from F to G is a family of morphisms  $\{\varphi_A: F(A) \rightarrow G(A)\}$  together with a family of 2-cells  $\{\varphi_f: G(f)\varphi_A \rightarrow \varphi_B F(f)\}$  as illustrated



satisfying obvious compatibility conditions. (The case where the  $\varphi_f$ 's are isomorphisms has been considered in [7] and [8], but we make no such restriction.) Given this notion of "natural transformation", it is reasonable and useful to inquire about the corresponding notion of "quasi-limit" or, more generally, "quasi-Kan extension".

Such a Kan extension was used in an essential way for the proof of the main result in [4, §9], but until now no justification has been given for calling the construction used there a "Kan extension". In the usual case, if  $S: \mathscr{A} \to \mathscr{B}$  is an ordinary functor and  $\mathscr{X}$  is a cocomplete category, then under appropriate hypotheses the functor

$$\mathscr{X}^{\mathbf{S}}:\mathscr{X}^{B}\to\mathscr{X}^{A}$$

is right adjoint to the (left) Kan extension  $\Sigma S: \mathscr{X}^A \to \mathscr{X}^B$ .  $\Sigma S$  can be constructed as follows: replace S by its associated factorization through an opfibration

$$\mathscr{A} \xrightarrow{Q_s} (S, \mathscr{B}) \xrightarrow{P_s} \mathscr{B}$$

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